In the following problems, make sure to write your arguments coherently in full sentences.

1. Let $V$ be a vector space over a field $F = \mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$. Suppose $v_1, v_2 \in V$ are linearly independent of each other. Show that vectors $v_1 + 2v_2$ and $2v_1 + 3v_2$ are linearly independent.

2. Show that the functions $y_1 = \sin x$, $y_2 = \cos x$ and $y_3 = \sin(2x)$ are linearly independent over $\mathbb{R}$.

3. Consider the set

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, a + d = 0 \right\}.$$ 

(a) Show that $V$ is a vector space over $\mathbb{R}$.
(b) Find a basis of $V$.

*Do the following problem for 6 bonus points.*

4. Are the functions $y_1 = \sin x$, $y_2 = \cos x$ and $y_3 = \sin(x + 1)$ linearly independent over $\mathbb{R}$? Verify your answer.