Some review problems for Final

1. Find the polynomial that fits the following points by Lagrange and Newton methods
   (a) \((-1, 1), (0, -1), (1, 1), (2, 0)\).
   (b) \((-1, 0), (0, -1), (1, 0), (0, 1)\).

2. Let \(f(x) = \frac{1}{2 + x^2}\). For evenly spaced sample points \(1 = x_1 < x_2 < \ldots < x_n = 2\), let \(P_n\) be the corresponding interpolation polynomial. Find \(n\) such that the integral \(\int_1^2 P_n(x)dx\) approximates the integral \(\int_1^2 f(x)dx\) with error not exceeding \(10^{-4}\).

3. We want to find an approximate value of the integral \(I = \int_2^4 \frac{1}{x^3 + 1}dx\). Let \(n\) be the number of equal subintervals of the interval \([2, 4]\).
   (a) For \(n = 5\), use right-point rule to approximate \(I\).
   (b) For \(n = 5\), use midpoint rule to approximate \(I\).
   (c) For \(n = 5\), use trapezoid rule to approximate \(I\).
   (d) For \(n = 5\), use Simpson rule to approximate \(I\).
   (e) How big should \(n\) be such that the approximate value of \(I\) by midpoint rule is under \(10^{-4}\)?
   (f) The same question as Part (e) but for Simpson rule.