1. Let $f(x) = xe^{-x^2}$.
   
   (a) Find the degree $2n + 1$ Taylor polynomial for $f(x)$, about the point $x_0 = 0$.
   
   Hint: use the identity with $t = -x^2$
   
   $$e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots + \frac{t^m}{m!} + R_m(t).$$
   
   (b) Bound the error in degree $2n + 1$ approximation for $|x| \leq 2$.
   
   Hint: use Lagrange theorem to bound the term $R_n(t)$ where $t = -x^2$.
   
   (c) Find $n$ so as to have $2n + 1$ Taylor approximation with error of at most $10^{-9}$ on $[-2, 2]$.

2. Convert the number $(101.011)_2$ from binary system to decimal system. (Make sure to show all your calculations, not just the final result.)

3. Convert the number $3.7$ from decimal system to binary system. (Make sure to show all your calculations, not just the final result.)

In the following problems, use the floating-point format described in Worksheet 10/4/2019 (handed in class, also posted on Canvas and the course website).

4. Do the following operations. Write your results in both floating-point format and decimal format. Make sure to show all your calculations, not just the final result.
   
   (a) $(1.001)_2 \times 2^2 + (1.101)_2 \times 2^4$
   
   (b) $(1.001)_2 \times 2^1 - (1.101)_2 \times 2^3$
   
   (c) $(1.001)_2 \times 2^7 + (1.101)_2 \times 2^7$
   
   (d) $(1.001)_2 \times 2^6 + (1.100)_2 \times 2^{-2}$

   What do you notice when adding these two numbers of quite different size?

5. What number does the bit sequence $10011011$ represent?

6. What is the smallest number greater than 1 that can be represented by floating-point format? Call this number $b$. The difference $\epsilon = b - 1$ is called the machine epsilon of this number format. Find $\epsilon$. 