Last time, we tried to evaluate \( \sqrt{8} \) with precision \( 10^{-9} \).

Our strategy was to approximate \( f(x) \), where \( f(x) = \sqrt{x} \), by Taylor approximation about \( x_0 = 9 \).

We found that

\[
\overline{f(x)} = p_n(x) + R_n(x) < 10^{-9} \text{ if } n > 10
\]

Choose \( n = 10 \).

\[
p_n(x) = f(x) + \sum_{k=1}^{n} \frac{f^{(k)}(x)}{k!} (x-x_0)^k
\]

\[
= 3 + \sum_{k=1}^{10} \frac{(x-9)^k}{k!} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \cdots \left( \frac{1}{2} \right) 9^{1-2k}
\]

To program this sum on Matlab, we use two "for" loops.

(see the Matlab file.)

Worksheet problems ...