Let us compute approximately the integral \( I = \int_1^3 x^2 \, dx \) by

- left-point rule (call the sum \( L_n \)),
- trapezoidal rule (call the sum \( T_n \)),

with \( n + 1 \) equally spaced sample points \( 1 = x_0 < \ldots < x_n = 3 \).

(a) Write \( L_n \) and \( T_n \) using sigma notation.

(b) Find \( n \) such that \( L_n \) approximates \( I \) with error not exceeding \( \epsilon = 10^{-4} \).

(c) The same question as in Part (b) for \( T_n \).
We know that
\[ |T - L_n| \leq \frac{M}{2} \frac{(b-a)^2}{n} \]
where \[ a = 1, \ b = 3 \]
\[ M = \max_{x \in [a, b]} |f'(x)| \]

We have \[ M = \max_{x \in [1, 3]} |2x| = \max_{x \in [1, 3]} 2x = 6 \]

Thus, \[ |T - L_n| \leq \frac{6}{2} \frac{(3-1)^2}{n} = \frac{8}{n} \]

To get \[ |T - L_n| < 10^{-4} \], we only need \( n \) such that
\[ \frac{8}{n} < 10^{-4} \]

This is equivalent to \( n > 80000 \). (A big number!)

The trapezoid rule will be mentioned in class soon.