1. Evaluate the following limits. You can use the command \texttt{Limit} in Mathematica to double check your results. Try the example: \texttt{Limit[1/(z + I), z \to \infty]}

(a) \[ \lim_{z \to i} \frac{z^3 + i}{z - i} \]

(b) \[ \lim_{z \to 0} \frac{\log(z + i) - \log i}{z} \]

(c) \[ \lim_{z \to 0} \frac{e^z - 1}{\log(z + 1)} \]

(d) \[ \lim_{z \to \infty} z \sin \left( \frac{1}{z} \right) \]

\textbf{Hint: change variable } w = 1/z.

2. Consider the function \( f(z) = \frac{z}{|z|} \).

(a) Write \( f(z) \) in complex standard form \( f = u + iv \). In other words, determine \( \text{Re } f(z) \) and \( \text{Im } f(z) \).

(b) Use Mathematica to plot \( u \) and \( v \). \textbf{Hint: use command \texttt{Plot3D}.}

(c) Find the limit of \( f(z) \) as \( z \) approaches 0 along each of the following paths:
   - the negative side of the real axis,
   - the positive side of the real axis,
   - the negative side of the imaginary axis,
   - the positive side of the imaginary axis.

(d) Find the limit of \( f(z) \) as \( z \) approaches \( \infty \) along each of the following paths:
   - the positive side of the real axis,
   - the positive side of the imaginary axis.

3. The principal argument is given by

\[
\text{Arg } z = \begin{cases} 
\arctan \left( \frac{y}{x} \right) & \text{if } x > 0, \\
\arccos \left( \frac{x}{\sqrt{x^2 + y^2}} \right) & \text{if } y > 0, \\
-\arccos \left( \frac{x}{\sqrt{x^2 + y^2}} \right) & \text{if } y < 0,
\end{cases}
\]

where \( z = x + iy \). Use Cauchy–Riemann theorem to verify that the function \( f(z) = \log z \) is holomorphic on \( \mathbb{C} \setminus \mathbb{R}_{\leq 0} \).

4. Verify that an antiderivative of \( f(z) = \log z \) on the region \( \mathbb{C} \setminus \mathbb{R}_{\leq 0} \) is \( F(z) = z \log z - z \).

5. Consider the function \( f(z) = \log z + \log(iz - i) \).

(a) Determine the region where \( f(z) \) is holomorphic.

(b) Determine all antiderivatives of \( f(z) \) on this region.

6. (Similar to Problem 4.5 on page 69 of the textbook) \textit{Use the definition of complex integration to integrate the following functions over the upper semicircle } \( C_2(0) \) \textit{oriented counter-clockwise.}

(a) \( f(z) = z + \bar{z} \)

(b) \( f(z) = z^2 - 2z + 3 \)

(c) \( f(z) = xy \)

(d) \( f(z) = \frac{1}{z^2} \)

\textbf{Hint: use de Moivre’s formula.