In Problem 3 and 4, you should use Mathematica to sketch the curves. You can use Mathematica to assist your arguments. Make sure to write the Mathematica code you use, give explanation and some comments on the graph.

1. Find the radius of convergence and the region of convergence of the following power series:

(a) \[ \sum_{n=1}^{\infty} \frac{2^n}{1 + 3^n} z^n \]

(b) \[ \sum_{n=1}^{\infty} \frac{z^n}{(\sqrt{3} + i)^n} \]

2. Find three different Laurent series representations (about 0) for the function

\[ f(z) = \frac{3}{-z^2 + 3 + 2} \]

3. Let \( \gamma \) be a path with parametrization

\[
\begin{cases}
  x(t) = 3 \cos t \cos 3t, \\
y(t) = 3 \sin t \cos 3t
\end{cases}
\quad t \in [0, 2\pi]
\]

and function

\[ f(z) = \frac{1}{(z^2 + 2z + 2)(z - 1)} \]

(a) Sketch \( \gamma \).

(b) Compute the exact value of \( \int_{\gamma} f(z)\,dz \). Write the result in complex standard form \( a + ib \).

\textit{Hint:} split \( \gamma \) into three simple curves. Pay attention to the orientation of each curve.

Use Cauchy’s Integral formula for each curve.

In Problem 4, similar treatment is done in the supplemental material “Evaluating complex integral by series” posted on the course website.

4. Let \( \gamma \) be a curve with parametrization

\[
\begin{cases}
x(t) = \sin t + \sin 2t, \\
y(t) = \cos t + \cos 5t
\end{cases}
\quad t \in [0, \pi]
\]

and function \( f(z) = \frac{e^z}{z^3} \).

(a) Sketch \( \gamma \).

(b) Find Laurent series representation of \( f(z) \) about 0.

(c) Use Part (b) to find an approximation for \( \int_{\gamma} f(z)\,dz \). Roundup to four digits after decimal point.