Some review problems for Final Exam

1. Compute the following limits. Distinguish between the case the limit is equal to $\infty$ and the case the limit does not exist.

(a) \[ \lim_{z \to 0} \frac{\sin z - z}{z^3} \]

(b) \[ \lim_{z \to -1+i} \frac{z + 1 - i}{z(z^2 + 2z + 2)} \]

(c) \[ \lim_{z \to i} \frac{e^{\pi z} + 1}{z - i} \]

(d) \[ \lim_{z \to 0} e^{i \text{Arg}(z^4)} \]

(e) \[ \lim_{z \to \infty} \frac{\text{Log} z}{z} \]

2. To each of the following functions, determine the region where it it continuous/ differentiable/ holomorphic. Find the derivative at $z = 1 + i$ if the function is differentiable at $i + 1$.

(a) \[ f(z) = \text{Log} \left( \frac{z+1}{z-1} \right) \]

(b) \[ f(z) = \text{Log}(iz^2) \]

(c) \[ f(z) = e^{iz} e^{iy} \]

3. Evaluate the complex integrals $\int_{\gamma} f(z)$ where $f$ and $\gamma$ are given as follows. Clearly mention the method/theorem you use.

(a) \[ f(z) = x^2 + iy \text{ and } \gamma \text{ is the part of the curve } x = \sqrt{y} \text{ from } y = 4 \text{ to } y = 1. \]

(b) \[ f(z) = \frac{1}{z + i} \text{ and } \gamma \text{ is the rectangle with vertices at } (1,0), (1,1), (-1,1), (-1,0) \text{ oriented in that order.} \]

(c) \[ f(z) = z\text{Log}(z^2 + 1) \text{ and } \gamma(t) = \frac{1}{2} e^{-it} \text{ where } t \in [0, \pi]. \]