Homework 2

1. Consider two following sums:

\[ S_n = \sum_{k=0}^{n-1} \frac{k}{n^2}, \quad S'_n = \sum_{k=1}^{n} \frac{k + 2}{n^2} \]

Given that \( \lim_{n \to \infty} S_n = \frac{1}{2} \), find \( \lim_{n \to \infty} S'_n \).

(Hint: compute the difference \( S'_n - S_n \).)

2. (Similar to Problems 21-24 of Section 5.2, page 359)

Each of the following sums is an approximation of the area of some region. Given that this is a region between graph of a function and the \( x \)-axis. Determine the corresponding function \( f \) and the interval. Use Mathematica to sketch the region.

(a)

\[ \sum_{k=1}^{n} \left( 1 + \frac{k^2}{n^2} \right) \frac{1}{n} \]

(b)

\[ \sum_{k=1}^{n} \left( 1 + \frac{k + 1}{n} \right) \frac{2}{n} \]

(c)

\[ \sum_{k=1}^{n} \ln \left( 1 + \frac{k + \frac{2}{3}}{n} \right) \frac{1}{n} \]

3. Practice finding antiderivatives: answer the first 10 homework questions (out of 21) of Section 4.9 on MyMathLab.

4. Practice using geometry to compute definite integrals: do Problems 25, 27, 29 of Section 5.2, page 359. Use Mathematica to sketch the graph of the integrands.