Some review problems for Midterm

1. Review Homework 1, 2, 3.

2. Review Worksheets 1 through 6.

3. Review Recitation worksheets.

4. Do Problem 1, Part (a) of Homework 4.

5. Consider the linear map $G : P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ given by $G(u) = xu' - u$. Is $G$ a monomorphism, epimorphism, isomorphism or none of them? Explain your answer.

6. Consider a linear map $f : M_{2 \times 2}(\mathbb{R}) \rightarrow M_{2 \times 2}(\mathbb{R})$ given by $f(A) = A^T + A$.
   (a) Find a matrix representation of $f$.
   (b) Consider
   $$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2 \times 2}(\mathbb{R}) : \quad a + 2b = 0 \right\}.$$ 
   Is $V$ invariant under $f$? Explain your answer.

7. Let
   $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -2 \\ 1 & 0 & 3 \end{bmatrix}.$$ 
   Put
   $$V_1 = \{ v \in \mathbb{R}^3 : \quad Av = v \},$$
   $$V_2 = \{ v \in \mathbb{R}^3 : \quad Av = 2v \},$$
   $$V_3 = \{ v \in \mathbb{R}^3 : \quad Av = 3v \}.$$ 
   Show that $V_1 \oplus V_2 \oplus V_3 = \mathbb{R}^3$. 
