1. Given an integer $n$ and numbers $\alpha_j$, $\beta_j$, $\gamma_j$ for $j = 1, 2, \ldots, n - 1$, how would you solve for $M_1, M_2, \ldots, M_n$ from the following system?

\[
\begin{align*}
\alpha_1 M_1 + \beta_1 M_2 &= \gamma_1, \\
\alpha_2 M_2 + \beta_2 M_3 &= \gamma_2 \\
\vdots & \quad \vdots \\
\alpha_{n-1} M_{n-1} + \beta_{n-1} M_n &= \gamma_{n-1}.
\end{align*}
\]

Can you write an algorithm to solve it?

Because there are $n$ unknowns and $n-1$ equations, we have the freedom to choose one unknown. Let us choose $M_1 = 0$. Then $M_2$ can be found from the first equation:

\[
M_2 = \frac{\gamma_1 - \alpha_1 M_1}{\beta_1}.
\]

Then $M_3$ can be found from the second equation

\[
M_3 = \frac{\gamma_2 - \alpha_2 M_2}{\beta_2}
\]

and so on. One can write the procedure as an algorithm as follows.

\[
M_1 = \text{a j (some specific number)}
\]

for $j = 2 : n$

\[
M_j = \frac{\gamma_{j-1} - \alpha_{j-1} M_{j-1}}{\beta_{j-1}}
\]

end
Matrix method:

One can solve the system using matrix method. With $M_i$ considered as being known, we can write the system as

\[
\begin{align*}
\beta_1 M_i &= \tau_i - \alpha_1 M_1, \\
\alpha_2 M_i + \beta_2 M_3 &= \tau_2, \\
&\vdots \\
\alpha_{n-1} M_{n-1} + \beta_{n-1} M_n &= \tau_{n-1}
\end{align*}
\]

In matrix form,

\[
\begin{bmatrix}
\beta_1 & 0 & \cdots & 0 \\
\alpha_2 & \beta_2 & 0 & \cdots \\
0 & \alpha_3 & \beta_3 & 0 & \cdots \\
\vdots & & & \ddots & \ddots \\
0 & 0 & \cdots & \alpha_{n-1} & \beta_{n-1}
\end{bmatrix}
\begin{bmatrix}
M_2 \\
M_3 \\
\vdots \\
M_n
\end{bmatrix}
= 
\begin{bmatrix}
\tau_2 \\
\tau_3 \\
\vdots \\
\tau_{n-1}
\end{bmatrix}
\]

This is the form $AX = b$. The solution is $X = A^{-1}b$.

One can enter matrix $A$ in Matlab by first initializing $A$ as an $(n-1) \times (n-1)$ matrix whose entries are all zeros.

\[A = \text{zeros}(n-1);\]

Then we fix the coefficients on the diagonal:

\[\text{for } j = 1:n-1 \\
A(j,j) = \beta_j;\]
\[\text{end}\]
Then we fix the coefficients on the sub-diagonal

\[ \text{for } j = l : n-2 \]

\[ A(j,j+l) = \alpha_{j+l,j} \]

end
2. How would you plot the following functions on the interval [0, 3] on the same graph using Matlab? (Use the command `hold on`)

\[ s_1(t) = -t^2 + t \]
\[ s_2(t) = t^2 - 3t + 2 \]
\[ s_3(t) = t^2 - 5t + 6. \]

See Lecture 24 (toward the end).