Name: ________________________________

Let us compute approximately the integral \( I = \int_1^3 x^2 \, dx \) by

- left-point rule (call the sum \( L_n \)),
- trapezoidal rule (call the sum \( T_n \)),

with \( n + 1 \) equally spaced sample points \( 1 = x_0 < \ldots < x_n = 3 \).

(a) Write \( L_n \) and \( T_n \) using sigma notation.

(b) Find \( n \) such that \( L_n \) approximates \( I \) with error not exceeding \( \varepsilon = 10^{-4} \).

(c) The same question as in Part (b) for \( T_n \).

\[
 \begin{align*}
(a) & \quad \text{The width of each subinterval is } \\
 & \quad h = \frac{3-1}{n} = \frac{2}{n}. \\
 & \quad \text{We have} \\
 & \quad x_0 = 1, \\
 & \quad x_1 = 1 + h, \\
 & \quad x_n = 1 + nh, \\
 & \quad \ldots \\
 & \quad x_n = 1 + nh. \\

& \quad \text{Thus, } x_k = 1 + kh = 1 + \frac{2k}{n}. \\

& \quad \text{The left-point Riemann sum is} \\
& \quad L_n = h f(x_0) + \ldots + h f(x_{n-1}) = h \sum_{k=0}^{n-1} f(x_k) \\
& \quad = h \sum_{k=0}^{n-1} x_k^2 \\
& \quad = h \sum_{k=0}^{n-1} \left( 1 + \frac{2k}{n} \right)^2 \\
\end{align*}
\]
\[ T_n = \frac{1}{2} h \left( f(x_0) + f(x_1) \right) + \cdots + \frac{1}{2} h \left( f(x_{n-1}) + f(x_n) \right) \]
\[ = \frac{1}{2} h \sum_{k=0}^{n-1} \left( f(x_k) + f(x_{k+1}) \right) \]
\[ = \frac{1}{2} h \sum_{k=0}^{n-1} \left[ \left( 1 + \frac{2k}{n} \right)^2 + \left( 1 + \frac{2(k+1)}{n} \right)^2 \right] \]

We will do part (b) and (c) next time.