

MTH 634 ALGEBRAIC TOPOLOGY

Homework Assignment I

Due October 14, 2011

Problem I.

- a) Describe (in detail) a cell complex structure for a closed, orientable surface $M_g = T^2 \# T^2 \# \cdots \# T^2$ of genus $g \geq 1$. (Here $\#$ indicates a connected sum.)
- b) Describe (in detail) a cell complex structure for a closed, non-orientable surface $N_g = \mathbb{R}P^2 \# \mathbb{R}P^2 \# \cdots \# \mathbb{R}P^2$ of genus $g \geq 1$.

Problem II. The join $X * Y$ of topological spaces X, Y is constructed from $X \times Y \times [0, 1]$ by prescribing the identifications $(x, y_1, 0) \sim (x, y_2, 0)$, $(x_1, y, 1) \sim (x_2, y, 1)$, where $x_1, x_2 \in X$, $y_1, y_2 \in Y$ are arbitrary. Thus the sets $X \times Y \times \{0\}$, $X \times Y \times \{1\}$ are collapsed to X and Y , respectively.

- a) Show that $S^1 * S^1 = S^3$.
- b) Show that in general, $S^m * S^n = S^{m+n+1}$.

Problem III.

- a) Construct a Δ -complex structure on the projective space $\mathbb{R}P^n$ as a quotient of a Δ -complex structure on S^n with the vertex set consisting of the intersections of S^n with the standard coordinate axes in \mathbb{R}^{n+1} .
- b) Compute the homology of the Δ -complex X obtained from the standard n -simplex Δ^n by identifying all faces of the same dimension $k < n$. (Thus X contains one k -simplex for all $k \leq n$.)

Problem IV. Hatcher, problem 17 on page 132.