1.) Implement the Sieve of Eratosthenes in Sage. Your function should accept as input an integer to determine the range of integers (starting from 2) on which you sieve. Run your function with upper bound 10000; check that you get the correct number of primes.

2.) Implement the Hawkins Probabilistic Sieve in Sage. The algorithm is as follows. Given the range bound $N$, let $S_1 = \{2, 3, \ldots, N\}$; let $h_1$ be the least element of $S_1$. Delete $h_1$ and each of the remaining elements of $S_1$ with probability $1/h_1$. Let $S_2$ be the result, and let $h_2$ be the least element of $S_2$ greater than $h_1$. Now delete each of the remaining elements of $S_2$ with probability $1/h_2$. Repeat as long as possible, creating a list of “Hawkins primes” $\{h_1, h_2, \ldots\}$.

One way to implement choice with probability $0 < p < 1$, is by use of $X = \text{GeneralDiscreteDistribution}([p-1/p, p])$ in combination with $X\.get\_random\_element()$.

Your procedure should accept as input the integer $N$. Average the number of “primes” you find over ten runs with $N = 10,000$, and compare with $\text{li}(10,000)$. Compare this average with $x/\log x$ for $x = 10,000$.

3.) Implement Cramér’s Probabilistic Primes in Sage. The algorithm is as follows. Given the range bound $N$, let $S = \{3, \ldots, N\}$; for each $n \in S$, declare $n$ to be prime with probability $1/\log n$.

Again, your procedure should accept as input an integer to determine the range of integers (starting from 3) from which a set of Cramér primes are extracted. Run your procedure with $N = 10,000$ ten times and take the average. Compare with $\text{li}(10,000)$.

4.) Plot the following partial sums in the plane, using upper bounds $N$ that work best for you. One approach is to convert the complex notation into $x, y$-coordinates and use polygon to get a collection of lines connecting the points.

a.) $\sum_{n=1}^{j} e^{2\pi in\sqrt{2}}$ for $1 \leq j \leq N$;

b.) $\sum_{n=1}^{j} e^{2\pi in \log(n)\sqrt{2}}$ for $1 \leq j \leq N$;

c.) $\sum_{n=1}^{j} e^{2\pi ip_n \sqrt{2}}$ for $1 \leq j \leq N$, where $p_n$ is the $n$-th prime.