1.) Compute the tensor products:
   a.) $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$.
   b.) $\mathbb{Q}[x] \otimes_{\mathbb{Q}} \mathbb{C}$.
   c.) $\mathbb{Q}/\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Q}/\mathbb{Z}$.

2.) Suppose that a rectangle in the plane $\mathbb{R}$ with sides parallel to the axes of length $a, b$ is subdivided into smaller rectangles $R_i$ with sides parallel to the axes, of lengths $a_i, b_i$ for some $i \in \{1, \ldots, n\}$. (Not all of the $R_i$ need reach the boundary of $R$.)

   Show that $a \otimes b = \sum_{i=1}^{n} a_i \otimes b_i$ in $\mathbb{R} \otimes_{\mathbb{Q}} \mathbb{R}$.

3.) Let $V$ and $W$ be vector spaces over a field $k$. Denote by $\text{BLF}(V)$ the vector space of $k$-bilinear functions $f : V \times V \to k$.

   a.) Show that there are natural linear maps:
       $\Phi : V^* \otimes W \to \mathcal{L}_k(V, W)$ such that $\Phi(\ell \otimes w)(v) = \ell(v) w$;
       $\Psi : V^* \otimes V^* \to \text{BLF}(V)$ such that $\Psi(\ell \otimes \ell')(v, v') = \ell(v) \ell(v')$;
       $T : V^* \otimes V \to k$ such that $T(\ell \otimes v) = \ell(v)$.

   b.) Prove that $\Phi$ and $\Psi$ are injective.

   c.) Prove that when $V, W$ are finite dimensional, then $\Phi$ and $\Psi$ are isomorphisms.

4.) Dummit and Foote, p. 455, #3.
   (Image of $\text{Sym}_2$)

5.) Dummit and Foote, p. 455, #10.
   (Identifying $\mathcal{A}^k(M)$)

6.) Dummit and Foote, p. 455, #13.
   (Symmetric and alternating 2-tensors)