

# Modification of the longwave and shortwave radiation budget in a simple climate model to include the direct radiative effect of aerosols

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The addition of the direct radiative effect of aerosols to the simple climate model of *Shell and Somerville* [2005] requires modification of the net solar (shortwave) heating ( $S$ ) and the net infrared (longwave) heating ( $I$ ) in the following basic model equations:

$$C_a \frac{\partial T_a}{\partial t} = 0 = S_a + I_a + F_s + F_l + D_a \quad (1)$$

$$C_s \frac{\partial T_s}{\partial t} = 0 = S_s + I_s - F_s - F_l + D_s \quad (2)$$

These equations are integrated forwards in time ( $t$ ) to determine the steady-state temperatures of the atmosphere ( $T_a$ ) and surface ( $T_s$ ) for each latitude. The subscripts  $a$  and  $s$  refer to atmospheric terms and surface terms respectively, and  $C$  is the heat capacity. The remaining heating and cooling terms, the sensible ( $F_s$ ) and latent ( $F_l$ ) heat fluxes from the surface to the atmosphere and the rearrangement of heat by ocean transport, midlatitude atmospheric eddies, and the Hadley circulation ( $D$ ), are not directly altered by the presence of aerosols, though they will of course change in response to a changing climate.

## 1. Shortwave radiation

In the aerosol-free version of the model, the shortwave heating is derived from the specified latitudinally-varying atmospheric absorptivity, reflectivity, and transmissivity, as well as surface albedo. Allowing for multiple reflections between the surface and the atmosphere, the model calculates the surface and atmospheric heating.

The addition of aerosols modifies these atmospheric radiative properties (absorptivity  $A$  and  $A^*$ , reflectivity  $R$  and  $R^*$ , and transmissivity  $T$  and  $T^*$ ; terms with no asterisks refer to the effect the atmosphere has on light from above, and terms with asterisks correspond to light from below.) We start by dividing the atmosphere into two aerosol-free layers

separated by an aerosol layer (Figure 1). We then use the adding method [Liou, 2002] to combine the aerosol layer with the layers of atmosphere above and below it.

To determine the shortwave radiative properties of the aerosol layer, we specify a broadband single scattering albedo ( $\omega$ ), asymmetry parameter ( $g$ ), and specific extinction cross section ( $B$ ). All three are constant with respect to time and latitude. We assume a gamma particle size distribution, which allows us to linearly relate the optical depth to the aerosol column loading:

$$\tau = \frac{3Q_{ext}m}{4\rho r_{eff}}$$

[Lacis and Mishchenko, 1995] where  $Q_{ext}$  is the dimensionless extinction efficiency factor,  $\rho$  is the density of the particles,  $r_{eff}$  is the effective radius of the aerosol, and  $m$  is the column mass per unit area ( $\text{g}/\text{m}^2$ ). We specify the specific extinction cross section,

$$B = \frac{3Q_{ext}}{4\rho r_{eff}},$$

with units of  $\text{m}^2/\text{g}$ , such that

$$\tau = Bm.$$

We use the  $\delta$ -Eddington approximation [Joseph et al., 1976] to determine the reflectivity, transmissivity, and absorptivity of the aerosol layer alone:

$$R_d = \omega'(1/2 - 3g'\mu_0/4)\frac{\tau'}{\mu_0} \quad (3)$$

$$T_d = 1 - R_d - (1 - \omega')\frac{\tau'}{\mu_0} \quad (4)$$

$$A_d = 1 - R_d - T_d = (1 - \omega')\frac{\tau'}{\mu_0} \quad (5)$$

$$\omega' = \frac{(1 - g^2)\omega}{1 - \omega g^2}, g' = \frac{g}{1 + g}, \tau' = \tau(1 - \omega g^2) \quad (6)$$

where  $\mu_0$  is the zenith angle.

We determine the optical properties of the atmospheric layers above ( $R_1$ ,  $T_1$ , and  $A_1$ ) and below ( $R_2$ ,  $T_2$ , and  $A_2$ ) the aerosol such that, when combined without the aerosol, they have the same optical properties as the original single atmospheric layer ( $R_a$ ,  $T_a$ , and  $A_a$ ):

$$R_a = R_1 + \frac{T_1 R_2 T_1}{1 - R_1 R_2} \quad (7)$$

$$R_a = R_2 + \frac{T_2 R_1 T_2}{1 - R_1 R_2} \quad (8)$$

$$T_a = \frac{T_1 T_2}{1 - R_1 R_2} \quad (9)$$

$R_a$  and  $T_a$  are specified as a function of latitude [see Section 2b in *Shell and Somerville*, 2005], and the reflectivity of the original layer is the same regardless of whether light is approaching from above or below. The ratio of  $R_1$  to  $R_2$  is proportional to their optical depths, which are determined from the aerosol height (given as the atmospheric pressure at the height of the aerosol layer):

$$R_1 = \frac{\tau_1}{\tau_2} R_2 = \frac{P_d}{P_0 - P_d} R_2 \quad (10)$$

where  $P_0$  is the surface pressure and  $P_d$  is the atmospheric pressure of the aerosol layer. Both are specified and constant with latitude. This distribution of reflectivity assumes that the reflecting particles are evenly spaced throughout the atmosphere. However, we expect that cloud droplets, the major reflecting particles, will be clustered at particular heights. This effect can be included in the model by adjusting Equation 10 appropriately.

$R_2$  is then one of the roots of

$$R_2^2 \left( -\frac{\tau_1}{\tau_2} + T_a^2 \frac{\tau_1}{\tau_2} \right) + R_2 \left( R_a + \frac{\tau_1}{\tau_2} R_a \right) - R_a^2 = 0 \quad (11)$$

Only one of these roots is valid ( $0 \leq R_2 \leq 1$ ) and results in valid values for the other optical properties, which are then calculated as follows:

$$T_2^2 = \frac{T_a^2(1 - R_1 R_2)R_2}{R_a - R_1} \quad (12)$$

$$T_1 = \frac{T_a}{T_2}(1 - R_1 R_2). \quad (13)$$

We next combine all three layers (the aerosol layer and two atmospheric layers), starting with the top two:

$$R_{top} = R_1 + \frac{T_1^2 R_d}{1 - R_1 R_d} \quad (14)$$

$$R_{top}^* = R_d + \frac{T_d^2 R_1}{1 - R_1 R_d} \quad (15)$$

$$T_{top} = \frac{T_1 T_d}{1 - R_1 R_d} \quad (16)$$

The top layer is combined with the bottom:

$$R = R_{top} + \frac{T_{top}^2 R_d}{1 - R_{top}^* R_2} \quad (17)$$

$$R^* = R_2 + \frac{T_2^2 R_{top}^*}{1 - R_{top}^* R_2} \quad (18)$$

$$T = T^* = \frac{T_{top} T_2}{1 - R_{top}^* R_2} \quad (19)$$

$$A = 1 - R - T \quad (20)$$

$$A^* = 1 - R^* - T^* \quad (21)$$

Although we first summed the top two layers, the result is the same if we start by summing the bottom two layers first. These values are then used as the atmospheric optical properties in Equations 5 through 7 of *Shell and Somerville* [2005].

The shortwave aerosol forcing is defined as the difference between the TOA or surface shortwave heating calculated using this method and the heating calculated without aerosols (i.e., using the equations described in *Shell and Somerville* [2005]).

## 2. Longwave radiation

The aerosol-free longwave budget includes emission and absorption by the surface and atmosphere. The atmospheric terms are influenced by a latitudinally-varying emissivity, which depends on the boundary layer temperature. Thus, the longwave calculation includes a simple water vapor feedback parameterization.

To determine the longwave aerosol forcing, we use the simple model developed by *Markowicz et al.* [2003]. This model assumes the effect of the aerosol is in the atmospheric window (8-12  $\mu\text{m}$ ) and that no gaseous or cloud-related absorption occurs in the window. In addition, the aerosol layer is treated as isothermal, and multiple scattering within the layer is neglected. Assuming the aerosol is at ambient temperature, the aerosol temperature,  $T_d$ , is based in the specified longwave height of the aerosol,  $H_d$ , compared to the representative height of the atmosphere,  $H_{mid}$ , and the lapse rate,  $lr$ :

$$T_d = T_a + lr \times (H_{mid} - H_d) \quad (22)$$

The surface and atmospheric forcing by the aerosol is

$$\Delta I_s = \tau_{ir} \sigma [0.5 T_s^4 \beta(T_s) \omega_{ir} (1 - g_{ir}) + T_d^4 \beta(T_d) (1 - \omega_{ir})] \quad (23)$$

$$\Delta I_a = \tau_{ir} \sigma (1 - \omega_{ir}) [T_s^4 \beta(T_s) - 2 T_d^4 \beta(T_d)] \quad (24)$$

where  $\omega_{ir}$  indicates a broadband optical property of the aerosol for the longwave part of the spectrum, and  $\beta$  is the percentage of total emitted radiation in the atmospheric window.

We use the cubic polynomial derived by *Pujol and North* [2002]:

$$\beta(T) = -0.737774 + 0.00670592 K^{-1} T - 1.39486 \times 10^{-5} K^{-2} T^2 + 9.02909 \times 10^{-9} K^{-3} T^3 \quad (25)$$

The first term of Equation 23 represents the longwave radiation emitted by the surface which is reflected by the aerosol layer back down to the surface. The second term corresponds to the emission of longwave radiation by the layer towards the surface. The atmospheric forcing (Equation 24) is composed of the absorption by aerosols of radiation emitted by the surface as well as the emission of radiation by the aerosol layer. The top of atmosphere (TOA) forcing due to the aerosol is simply the sum of the surface and atmospheric forcings.

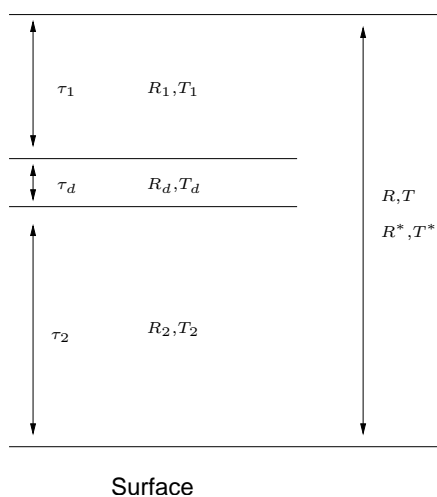
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Variable	Description	Value	Units
$\omega$	SW single scattering albedo of dust	0.97	-
$g$	SW asymmetry parameter of dust	0.78	-
$B$	SW specific extinction cross section of dust	1.64	m <sup>2</sup> /g
$P_d$	SW dust layer pressure	780	mb
$\omega_{ir}$	LW single scattering albedo of dust	0.5	-
$g_{ir}$	LW asymmetry parameter of dust	0.61	-
$B_{ir}$	LW specific extinction cross section of dust	0.4	m <sup>2</sup> /g
$h_d$	LW dust layer height	3	km
$d$	Dust concentration		$\mu\text{g}/\text{kg}$
$\mu_0$	Zenith angle		-
$\Delta I_s$	Surface LW dust forcing		W/m <sup>2</sup>
$\Delta I_a$	Atm LW dust forcing		W/m <sup>2</sup>
$R_d$	Reflection of dust layer		-
$T_d$	Transmission of dust layer		-
$A_d$	Absorption of dust layer		-
$R$	Reflection of atm to light from above		-
$R^*$	Reflection of atm to light from below		-
$T$	Transmission of atm to light from above		-
$T^*$	Transmission of atm to light from below		-
$A$	Absorption of atm to light from above		-
$A^*$	Absorption of atm to light from below		-
$\frac{\tau_1}{\tau_2}$	SW optical depth fraction		-
$\tau$	SW optical depth of dust		-
$\tau_{ir}$	LW optical depth of dust		-

**Table 1.** Model variables for the default case of mineral dust. Values are listed for those that are constant with latitude. See Table 1 in *Shell and Somerville* [2005] for model variables not related to aerosols.



**Figure 1.** Shortwave optical properties for an atmosphere with an aerosol layer. Refer to Table 1 for definitions of variables.