Math 644 Homework 4  
Due Friday March 8

Practice Problems

1. (Section 9.1, problems #4, #5)
   (a) Prove that the ideals \((x)\) and \((x, y)\) are prime in \(\mathbb{Q}[x, y]\) but only the latter ideal is maximal.
   (b) Prove that the ideals \((x, y)\) and \((2, x, y)\) are prime in \(\mathbb{Z}[x, y]\) but only the latter ideal is maximal.

2. (Section 9.4, problems #1, #2)
   Determine (prove) whether the following polynomials are irreducible in the indicated ring. We write \(\mathbb{F}_p := \mathbb{Z}/p\mathbb{Z}\).
   (a) \(x^2 + x + 1\) in \(\mathbb{F}_2[x]\)
   (b) \(x^4 + 1\) in \(\mathbb{F}_5[x]\)
   (c) \(x^4 + 10x^2 + 1\) in \(\mathbb{Z}[x]\)
   (d) \(x^4 - 4x^3 + 6\) in \(\mathbb{Z}[x]\)
   (e) \(x^6 + 30x^5 - 15x^3 + 6x - 120\) in \(\mathbb{Z}[x]\)

Hand-In Problems

1. (Section 8.2, problem #3)
   Prove that a quotient of a PID by a prime ideal is again a PID.

2. (Section 9.2, # 3; Section 9.4, problem #7)
   (a) Let \(f(x)\) be a polynomial in \(F[x]\). Prove that \(F[x]/(f(x))\) is a field if and only if \(f(x)\) is irreducible.
   (b) Prove that \(\mathbb{R}[x]/(x^2 + 1)\) is a field and is isomorphic to \(\mathbb{C}\).