Math 645 Homework 2
Due Friday May 3

Practice Problems

1. (Section 13.1, problem #3)
   Show that $x^3 + x + 1$ is irreducible over $\mathbb{F}_2$, and let $\theta$ be a root. Compute the powers of $\theta$ in $\mathbb{F}_2(\theta)$.

2. (Section 13.2, problem #10)
   Determine the degree of the extension $\mathbb{Q}(\sqrt{3} + 2\sqrt{2})$ over $\mathbb{Q}$.

Hand-In Problems

1. (Section 13.2, problem #7)
   (a) Prove that $\mathbb{Q}(\sqrt{2} + \sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$.
   (b) Conclude that $[\mathbb{Q}(\sqrt{2} + \sqrt{3}) : \mathbb{Q}] = 4$.
   (c) Find an irreducible polynomial satisfied by $\sqrt{2} + \sqrt{3}$.

2. (Section 13.4, problem #1)
   Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 - 2$. 