Math 649 Homework 3
Due Friday March 8

1. Prove that for any fixed $s_0 \in \{0, 1, \ldots, p-2\}$, the set of nonnegative integers congruent to $s_0$ modulo $(p-1)$ is dense in $\mathbb{Z}_p$, i.e., any number in $\mathbb{Z}_p$ can be approximated by such numbers.

2. Let $f : \mathbb{Z}_p \rightarrow \mathbb{Q}_p$ be the locally constant function defined by taking $x \in \mathbb{Z}_p$ to the first digit in its $p$-adic expansion. Find $\int f \mu$ in each of the following cases:
   
   (a) $\mu = \mu_{\text{Haar}}$,
   (b) $\mu = \mu_{\alpha}$ (Dirac distribution),
   (c) $\mu = \mu_{\text{Mazur}}$.

3. Prove the following facts about the Bernoulli polynomials $B_k(x)$.
   
   (a) $B_k(x) = \sum_{i=0}^{k} \binom{k}{i} B_i x^{k-i}$, and in particular $B_k(0) = B_k$.
   (b) $\int_0^1 B_k(x) dx = \begin{cases} 1 & \text{if } k = 0, \\ 0 & \text{otherwise}. \end{cases}$
   (c) $\frac{d}{dx} B_k(x) = k B_{k-1}(x)$. 