Recall the following from class.

**Fact.** Let $p$ be prime and $\alpha \geq 1$. Then

$$\tau(p^{\alpha+1}) = \tau(p)\tau(p^{\alpha}) - p^{11}\tau(p^{\alpha-1}).$$

Homework Problems.

1. Show that for any prime $p$,

$$\sum_{n \geq 0} \tau(p^n)x^n = \frac{1}{1 - \tau(p)x + p^{11}x^2}.\]

[Hint: Multiply both sides by the denominator of the right hand side and compare coefficients.]

2. Prove that for $n > 1$,

$$(1 - n)\tau(n) = 24\sum_{j=1}^{n-1} \sigma_1(j)\tau(n - j),$$

and use this to deduce that $\tau(n) \equiv 0 \pmod{8}$ for any even $n$.

[Hint: From the proof of the infinite product representation for $\Delta(z)$, can show that $\frac{1}{2\pi i}\Delta'(z) = \Delta(z)E_2(z)$, and compare coefficients.]