1. Show that \( C = \{ (0,0,0), (1,1,1) \} \) is an \( \mathbb{F}_2 \)-vector subspace of \( \mathbb{F}_2^3 \).
   
   (a) What is its dimension?
   (b) Give a linear transformation from \( \mathbb{F}_2 \) to \( \mathbb{F}_2^3 \) whose image is \( C \).
   (c) Express this as right multiplication by a matrix. What is the rank of the matrix?
   (d) What is the minimum distance of \( C \)?

2. Use reduction followed by column permutation to place the following \( \mathbb{F}_3 \)-matrix into systematic form.

\[
\begin{pmatrix}
1 & 0 & 1 & 2 \\
1 & 1 & 2 & 1 \\
2 & 1 & 0 & 2 \\
\end{pmatrix}
\]

3. Give the length, size, and dimension for the binary linear code with generator matrix

\[
G = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 \\
\end{pmatrix}
\]

4. For a finite field \( \mathbb{F} \) and positive integer \( n \), show that for each \( d \leq n \) there exists an \( [n,1] \) \( \mathbb{F} \)-linear code with minimum distance \( d \).

5. Go to your existing SageCloud project called (Your Name) Coding Theory Homework, to which you have invited me (swisherh@gmail.com) as a collaborator from HW1. Start a new Sage Worksheet and call it HW2. In this worksheet, write a function called HamW(x), which takes as input a vector or list x, and returns the Hamming weight of the vector.

Recall, if you are not familiar with python syntax, I highly recommend the Codecademy course on python (https://www.codecademy.com/learn/python), it’s really fun and helpful!