

## Homework # 6 - Wed Aug 12 2009

This will be last homework assignment. Please make sure to turn in your work by Noon of Wednesday August 12th so I can return your work at the last meeting (and final exam date) of Friday August 14th.

**Problem 1:** Recall that given a vector norm  $|X|$  the operator norm of a matrix  $A$  is given by  $|A| = \max_{|X|=1} |AX|$ .

Part I: Show that  $|I| = 1$  for any the vector norm used. As always,  $I$  is the identity matrix.

Part II: Give at least one example to show that if  $A$  is invertible, the condition number of  $A$ ,  $\kappa(A) = |A||A^{-1}|$  satisfies

$$\kappa(A) > 1.$$

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**Problem 2:** Assume that a computer can perform  $10^6$  multiplications per second. Estimate the time that it would take to evaluate the determinant of a  $100 \times 100$  matrix  $M$ . You should not assume that you know the  $LU$  factorization of  $M$  but that you obtain it in approximately  $(1/3)n^3$  operations (See Table 6.1 in page 277 of the text.)

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**Problem 3:** Let  $a = 10^{-10}$  and consider the system

$$\begin{pmatrix} 1 & 1 \\ a & 0 \\ 0 & a \end{pmatrix} X = \begin{pmatrix} -a \\ 1+a \\ 1-a \end{pmatrix}$$

Part I: Set up the normal equations  $A^T A X = A^T B$ . Verify by hand that  $X = [1, -1]^T$  is the solution of the normal equations, corresponding to the least square solution.

Part II: Try computing the least square solution using MATLAB with the normal equations. Compare the solution with the exact solution obtained in Part I.

Part III: Use MATLAB `qr` command to compute the least square solution by the QR method and compare again with the solution obtained in Part I.

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**Problem 4:** Consider the non-linear system

$$x^2 + y^2 = 4, \quad x^2 - y^2 = 1,$$

Part I: Check that the exact solutions are the four points with coordinates  $x = \pm\sqrt{2.5}$ ,  $y = \pm\sqrt{1.5}$ .

Part II: Using as initial guess  $x_0 = 1.6$ ,  $y_0 = 1.2$  perform one iteration using Newton's method.

Part III: Write a MATLAB program to implement Newton's method.

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