MIDTERM EXAM

INSTRUCTIONS: This exam has a total of 120 points. Point values for each question are listed to the right of each question number. All students will answer questions 1-4. In addition, ECON 424 students will answer question 5A while ECON 524 students will answer 5B. Good Luck!

1. (34 points) According to Keynes, aggregate consumption is determined by aggregate income, but the classical economists claim that consumption is inversely related to interest rates. Using observations on U.S. consumption expenditures (abbreviated C; in billions of 1982 dollars), disposable personal income (M; in billions of 1982 dollars) and the real interest rate (R) for the years 1955-86, the following models are estimated. Standard errors are in parentheses.

\[
\begin{align*}
(1) & \quad \hat{c}_t = -3.77 + 0.91 M_t \\
& \quad (13.5) \quad (0.008) \\
(2) & \quad \hat{c}_t = 1370.7 + 59.86 R_t \\
& \quad (115.4) \quad (29.5)
\end{align*}
\]

a. Identify and interpret the following parameter estimates verbally:
   i. \( b_1 \) in equation (2)
   ii. \( b_2 \) in equation (1)
   iii. \( b_2 \) in equation (2)

b. Do the signs of the parameter estimates in (2) support the Classical theory?

c. Test whether the parameter estimate on \( R_t \) in equation (2) is significantly different from zero at the 5 percent level of significance using a two-tailed test.

d. For the Keynes' hypothesis, compute and interpret a confidence interval estimate for \( b_2 \), the marginal propensity to consume.

e. Based on your results, what do you conclude about the two hypotheses?

f. If disposable personal income in the U.S. was 100 billion dollars (in 1982 $), what would be the expected consumption level?

2. (8 points) Suppose that political analysts predict that 45 percent of voters will vote for Candidate Can-do based on regression estimates using data on 1,000 voters. The stated “margin of error” is ±3 percentage points, so that the analysts expect the percent of voters favoring the candidate to lie between 42 and 48. Assuming that they are using a 5 percent significance level to construct their prediction interval, calculate the standard error of the forecast \( \text{se}(f) \).
3. (28 points) This question applies to the following computer output.

Model: MODEL1
Dependent Variable: Y

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1</td>
<td>25221.22299</td>
<td>25221.22299</td>
<td>17.647</td>
<td>0.0002</td>
</tr>
<tr>
<td>Error</td>
<td>38</td>
<td>54311.33145</td>
<td>1429.24556</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Total</td>
<td>39</td>
<td>79532.55444</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Root MSE 37.80536  R-square 0.3171
Dep Mean 130.31300  Adj R-sq 0.2991
C.V. 29.01120

Parameter Estimates

| Variable | DF | Parameter | Standard Error | T for H0: Parameter=0 | Prob > |T| |
|----------|----|-----------|----------------|------------------------|---------|----|
| INTERCEP | 1  | 40.767556 | 22.13865442    | 1.841                  | 0.0734  |
| X        | 1  | 0.128289  | 0.03053925     | 4.201                  | 0.0002  |

a. Write down the estimated sample regression equation.
b. Graph the estimated regression line. Be sure to label the axes and indicate the intercept and slope terms.
c. Interpret the value of $R^2$ verbally.
d. Define $R^2$ mathematically, and demonstrate that the $R^2$ value on the output is consistent with the other figures in the Analysis of Variance table.
e. Using only the Prob values listed in the table above, determine whether or not the intercept and slope coefficient estimates are significantly different from zero at the 5 percent level of significance.

4. (25 points)
a. Other than goodness-of-fit, explain why we use the least-squares estimator to estimate economic relationships.
b. Under what assumptions does your answer in part (a) hold? List the assumptions.

For ECON 424 Students

5A. (25 points)
a. Define the variance of a random variable, $x$, in terms of expected values.
b. Show that the variance of $x = E(x^2) - [E(x)]^2$.
c. Define the covariance of two random variables $x$ and $y$.
d. Show that the cov $(x, y) = E(xy) - E(x) E(y)$. 
5B. (25 points) Suppose that you believed that the true model of interest is given by:

\[ y_t = \beta_2 x_t + e_t \]  

You estimate the model using the least-squares estimator (which we derived in the homework),

\[ \hat{\beta}_2 = \frac{\sum x_t y_t}{\sum x_t^2} \]

a. Assuming that the true model is given by (3) and that the assumptions of the Gauss-Markov Theorem hold, is the estimator in (4) unbiased?

b. Suppose that you are mistaken, and the true model is actually:

\[ y_t = \beta_1 + \beta_2 x_t + e_t \]

Is the estimator given by (4) unbiased? (Do not answer this question by comparing it to the usual least-squares estimator.)

c. Consider a different modeling mistake— the true model is (3) but you estimate (5). What do you think your computer output would show regarding the parameter estimates and t-ratios? (No need for a mathematical derivation here, just an intuitive answer.)