Instructions: This homework assignment is worth 50 points and each question is equally weighted. You are encouraged to work together with other students, but please write up your answers independently and submit an answer only if you understand it.

1. Assume a competitive market where each firm faces the following long-run total cost function.
   \[(\text{Total Cost Function}) \quad TC = 35q - 5q^2 + q^{3/4}.\]
The market demand function is:
   \[(\text{Demand}) \quad Q = 1210 - p,\]
where \(Q\) is the sum of each firm’s output \((q)\).
   A. Determine the long-run equilibrium price and output for the representative firm.
   B. Determine the long-run equilibrium level of output in the market.
   C. Determine the long-run equilibrium number of firms.

2. Assume a competitive market with the following inverse demand and supply functions.
   \[(\text{Inverse Demand}) \quad p = 35 - 2Q\]
   \[(\text{Inverse Supply}) \quad p = Q^2\]
where \(p\) is price and \(Q\) is market output.
   A. Determine the equilibrium price and quantity.
   B. If social welfare is defined as consumer plus producer surplus, define and use this function to determine the socially optimal price and output.
   C. What is consumer surplus and producer surplus at the social optimum.

3. Consider a market with a market demand function and firm(s) that face the total cost function described below.
   \[(\text{Inverse Demand}) \quad p = a - b Q\]
   \[(\text{Total Cost}) \quad TC = c q\]
where \(p\) is price, \(q\) is firm output, and \(Q\) is market output. Parameters \(a\), \(b\), and \(c\) are positive and \(a-c > 0\).
   A. Assuming a perfectly competitive market with constant costs, derive the market equilibrium price and output \((Q)\).
   B. Assuming a monopoly market, derive the firm’s optimal price and output.
   C. Explain how profits and the level of efficiency change as the market goes from perfectly competitive to monopoly.
4. Assume a monopolist is able to effectively price discriminate in two markets (A and B). The firm’s demand and cost conditions are described below.

\[
\begin{align*}
&\text{Inverse Demand in Markets A and B} \quad p_A = 24 - q_A \quad \text{and} \quad p_B = 12 - 2q_B \\
&\text{(Total Cost)} \quad TC = 2q
\end{align*}
\]

A. Derive the firm’s optimal price and output levels in markets A and B.
B. Show that the equilibrium price is higher in the market with the demand that is relatively more inelastic.

5. Assume a monopolist sells its output in a single market but production takes place in two plants (A and B). The firm’s demand and cost conditions are described below.

\[
\begin{align*}
&\text{Inverse Demand} \quad p = 12 - q \\
&\text{(Total Cost for Plants A and B)} \quad TC_A = \left(q_A^2\right)/2 \quad \text{and} \quad TC_B = q_B^2
\end{align*}
\]

where \( q = q_A + q_B \).

A. Derive the firm’s optimal price and output levels for each plant.
B. Show that the firm produces more output in the more efficient plant.

6. Assume a monopolist has two choice variables, price (p) and quality (z). The firm’s demand and cost functions are described below:

\[
\begin{align*}
&\text{(Demand)} \quad q = a - p + z \\
&\text{(Total Cost)} \quad TC = z^2,
\end{align*}
\]

where parameter “a” is a positive constant.

A. Suppose quality is fixed at 0. Determine the firm’s profit maximizing price, output, and level of profits.
B. Now suppose the firm can choose both p and z. Determine the firm’s profit maximizing price, output, and level of profits.
C. Is the firm better off with control of product quality? Explain.

7. Suppose the following is a differentiable function in y:

\[
x + y^3 = y^5 - x^2 + 2y
\]

Find \( \frac{dy}{dx} \).

8. Consider a competitive market with the following demand and supply functions:

\[
\begin{align*}
&\text{(Demand)} \quad q_D = D(p, m) \\
&\text{(Supply)} \quad q_S = S(p, t)
\end{align*}
\]

where \( q_D \) is quantity demanded, \( q_S \) is quantity supplied, \( p \) is price, \( m \) is consumer income, and \( t \) is a per-unit tax. Assume positively sloped supply and negatively sloped demand functions.

A. Use the implicit-function theorem to determine the impact of a change in \( m \) on the equilibrium price.
B. Use the implicit-function theorem to determine the impact of a change in \( t \) on the equilibrium price.
C. Under what conditions would it be \textbf{inappropriate} to use the implicit-function? Explain.
9. Assume a firm uses just one input (x) to produce output and operates in competitive output and input markets. The firm’s profit function is given below.

\[ \pi = pq(x) - w x, \]

where \( \pi \) is profit and \( w \) is the price of the input.

A. If the firm’s goal is to choose its profit maximizing level of x, determine the firm’s first and second order conditions for this problem. Provide an economic interpretation of these conditions.

B. Use the implicit-function theorem to determine the impact of a change in \( w \) on the optimal x.

10. Consider the following normal-form games.

<table>
<thead>
<tr>
<th>Game 10A</th>
<th></th>
<th>Game 10B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2</td>
<td></td>
<td>Player 2</td>
</tr>
<tr>
<td>L</td>
<td>R</td>
<td>X</td>
</tr>
<tr>
<td>U 3, 3</td>
<td>2, 0</td>
<td>A 8, 3</td>
</tr>
<tr>
<td>Player 1</td>
<td></td>
<td>0, 4</td>
</tr>
<tr>
<td>D 4, 1</td>
<td>8, 1</td>
<td>4, 4</td>
</tr>
<tr>
<td>C 3, 7</td>
<td></td>
<td>0, 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2, 0</td>
</tr>
</tbody>
</table>

A. Identify all dominated strategies in games 10A and 10B.

B. Determine all dominant-strategy, iterated-dominant strategy, and Nash equilibria in games 10A and 10B.

11. Assume Pat and Chris each own a soft-drink stand that is mobile and serve customers along a stretch of beach that is segmented into 9 regions. Each region has the same area and an equal number of customers that are uniformly dispersed. Pat and Chris charge the same price for a soft drink. In a given period, sales from each region will be $50, which would generate a total profit of $10 per region. Each vender can locate his or her stand in the center of any of the 1-9 regions, and customers walk to the nearest stand to purchase soft drinks. If customers in a given region are indifferent between the two stands (i.e., they are the same distance away), assume that half go to one stand and half go to the other. If each vender wants to maximize their own individual profit, determine their Nash equilibrium locations. Explain.
12. Consider the following normal-form game.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0,0</td>
<td>0,5</td>
<td>0,0</td>
</tr>
<tr>
<td>B</td>
<td>5,0</td>
<td>0,0</td>
<td>-5,0</td>
</tr>
<tr>
<td>C</td>
<td>0,0</td>
<td>0,-5</td>
<td>-5,-5</td>
</tr>
</tbody>
</table>

A. Identify all iterated-dominant equilibria.
B. Identify all Nash equilibria.

13. Consider a triopoly market, where firms 1, 2, and 3 have perfect and complete information and compete by simultaneously choosing output. The demand and cost functions are defined as follows.

\[
\text{(Inverse Demand)} \quad p = a - (q_1 + q_2 + q_3) \\
\text{(Total Cost)} \quad TC_1 = c_1 q_1, \quad TC_2 = TC_3 = 0,
\]

where parameters \( a \) and \( c_1 \) are positive and \( a > c_1 \).

A. Determine the Nash equilibrium output, price and profit levels for each firm.
B. How does a change in \( c_1 \) affect the equilibrium?

14. Consider a triopoly market, where firms 1, 2, and 3 have perfect and complete information and compete by simultaneously choosing prices. The demand and cost functions are defined as follows.

\[
\text{(Firm i’s Demand)} \quad q_i = a - p_i + d(p_j + p_k) \\
\text{(Total Cost)} \quad TC_i = 0,
\]

where \( a \) and \( d \) are parameters and \( a > 0 \). Note that subscript \( i \) represents firms 1, 2, or 3 and subscripts \( j \) and \( k \) represent \( i \)’s other rivals.

A. Determine the Nash equilibrium output, price and profit levels for each firm.
B. What restrictions need to be placed on \( d \) to assure: (1) that firms participate in the game and (2) that the products are substitutes?
C. How does a change in \( d \) affect the equilibrium?
15. Assume two firms (1 and 2) compete in marketing and have a choice of a low (L), medium (M), or high (H) expense marketing campaign. If their payoffs are described below, find all pure- and mixed-strategy Nash equilibria to this game.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0, 0</td>
<td>-1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>M</td>
<td>1, -1</td>
<td>-2, -2</td>
<td>-3, -2</td>
</tr>
<tr>
<td>H</td>
<td>0, 0</td>
<td>-2, -3</td>
<td>-3, -3</td>
</tr>
</tbody>
</table>

Firm 2