Heat Pulse Charging: Nonlinear Regression Fit

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December 2008

Line Charge Thermal Model (Bristow 94, page 449)

$$T(t) = T_0 + \frac{-Q'}{4\pi\kappa}\mathrm{Ei}\left(\frac{-r^2}{4\kappa t}\right) \tag{1}$$

Q': is the source strength per unit time ($m^2\ ^\circ\mathrm{C} s^{-1}$)

κ: thermal diffusivity $m^2 s^{-1}$

r: 'effective' radius (meters)

Note: $\mathrm{Ei}(-x) = -\mathrm{E}_1(x)$

Ball park figures
(Q' , κ, r) $=$ [1.287e-5 m^2degCs^{-1}, 5e-7 $m^2 s^{-1}$, 1e-3 meters]

Nonlinear fit to parameters: Fig 1 upper curve

Need starting values and bounds (min, max) for Q', κ, and r ?

Starting values (Q', κ, r) = [1.287e-5, 5e-7, 1e-4] – Q is set fixed to estimate.

Here is the fit for approx $(Q', \kappa, r) =$
(0.000012870000000 0.000000227403477 0.001213448506939) :

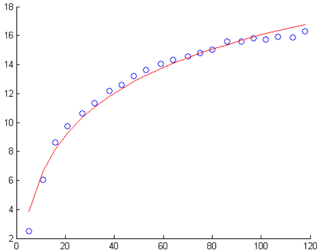


Figure: Fit with Q = 1.287e-5, Upper

So 'r' (effective radius) is approximately 1 mm.

Nonlinear fit to parameters: Lower curve

Starting values (Q', κ, r): [1.083e-5, 5e-7, 1e-4]
(Q', κ, r) = (1.0e-003)*(0.01083000000000 0.000310406551974 0.952919764737804)

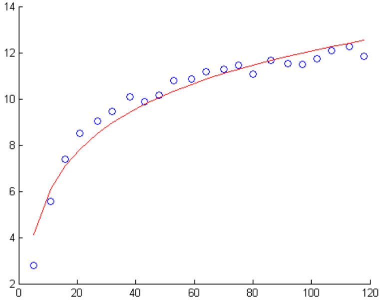


Figure: Fit with Q = 1.083e-5, Lower

So 'r' is approximately 1 mm again.

Method used to estimate Q'

Well if we know the soil moisture content (θ) at any location, we can estimate ρc as follow:

$$\rho c = (\rho c)_s + (\rho c)_w \theta_v$$

where $(\rho c)_s$ = volumetric heat capacity for soil

$(\rho c)_w$ = volumetric heat capacity for water

ρ_{sand} = 1.62 g/cm^3 (measured in the sand column)

c_s the specific heat capacity of soil solid = 0.83 j g^{-1} °C^{-1} (estimated for quartz sand)

c_w the specific heat capacity of water = 4.186 j g^{-1} °C^{-1} (at 15 °C)

ρ_w = .9991 g/cm^3 (at 15 °C)

If you plug in the water content for the 2 datasets you have you will get:

ρc = 1.554 MJ/m^3 °C for θ = 0.05 m^3/m^3

and ρc = 1.846 MJ/m^3 °C for θ = 0.12 m^3/m^3

Knowing that Q'=q/ρc and q = 20 J m^{-1}s^{-1} we can calculate Q' for each water content.

Thus, we can use the calculated Q' to find r for the two datasets and see if it is changing with moisture content.

For the thermal diffusivity, my best estimate is that it can vary from 1 x10^{-7} m^2 s^{-1} to 1 x10^{-6} m^2 s^{-1}

Q' estimate

Chadi Paper, Figure 1, 'Upper Curve':
For $\theta = 0.05$ m3/m3 $--->$ Q=20/1.544e6 = 1.287e-5 m2 C s-1

Chadi Paper, Figure 1, 'Lower Curve':
For $\theta = 0.12$ m3/m3 $--->$ Q=20/1.846e6 = 1.083e-5 m2 C s-1

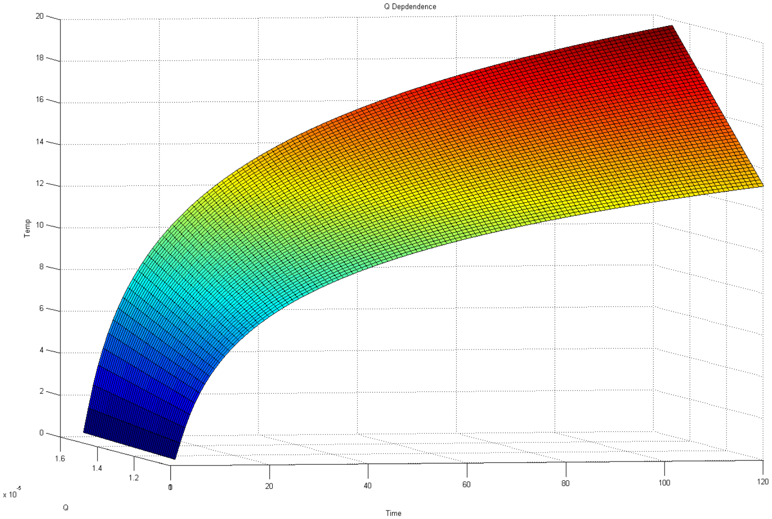
Thermal Integral Method Solution

If time of heat pulse is 'fixed', call it t_f, than the 'thermal integral' can be approximated by

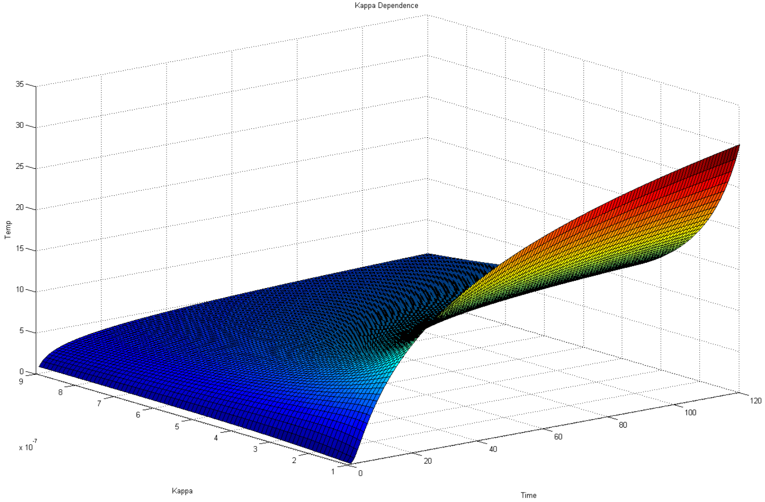
$$\int_0^{t_f} \Delta T dt = \frac{Q'}{4\pi\kappa}[(t_f - t_c) \cdot \ln(t_f - t_c) + d \cdot t_f + A]$$

for constants t_c, d, and A.

Q Dependence



Kappa Dependence



Radius Dependence