Towards an understanding of form drag at Three Tree Point (TTP)

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Dissipation in the ocean

3,700,000 MW: power put into ocean by tides

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2,600,000 MW: dissipation rate in the shallow seas

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0.72 MW: estimated work rate at Three Tree Point

Motivating questions

> How much energy is dissipated through form drag at TTP?

> What physical processes — such as internal waves and eddies — contribute to energy dissipation at TTP?

> Can form drag be parameterized for all ocean topography?
Form drag and mixing

Form drag:

\[ D_{\text{form}} = - \int_{A_0} \rho_B \frac{\partial h}{\partial x} \, dA_0 \]

- Streamlined body: small form drag
- Blunt body: large form drag

Pressure: $U_0$
Form drag and mixing

Form drag causes:
- internal wave generation
- eddy generation
- local mixing and dissipation

\[ D_{\text{form}} = - \int_{A_0} \rho_B \frac{\partial h}{\partial x} \, dA_0 \]

Energy conversion \[ = D_{\text{form}} U \]
Calculating form drag

momentum equation:

\[
\frac{Du}{Dt} + \mathbf{f} \mathbf{k} \times \mathbf{u} = -\frac{\nabla p}{\rho_0} - \frac{\hat{\mathbf{k}} g \rho}{\rho_0} + \nu \nabla^2 \mathbf{u}
\]

take the x-component of a volume integral:

\[
i \cdot \int_{V} \left\{ \frac{Du}{Dt} + f \hat{\mathbf{k}} \times \mathbf{u} = -\frac{\nabla p}{\rho_0} - \frac{\hat{\mathbf{k}} g \rho}{\rho_0} + \nu \nabla^2 \mathbf{u} \right\} dV
\]

\[
\int_{V} u_{i} dV + \int_{A} u u_{n} dA = f \int_{V} v dV = -\int_{A} \frac{p}{\rho_0} \mathbf{i} \cdot \mathbf{n} dA + \int_{A} \nu \frac{\partial u}{\partial n} dA
\]

form drag arises from this term

MacCready et al., 2003
Calculating form drag

The pressure terms becomes:

\[ - \int_A \frac{p}{\rho_0} \hat{i} \cdot \hat{n} dA = - \int_{\text{Asi des}} \frac{p}{\rho_0} \hat{i} \cdot \hat{n} dA - \int_{A_0} \frac{p_B}{\rho_0} h_x dA \]

Form Drag

MacCready et al., 2003
Motivation
Puget Sound, WA

Previous work
- Measured the internal form drag
  Edwards et al., 2004
- Measured the external form drag
  McCabe et al., 2006

Our goal
- Measure the total form drag with bottom pressure sensors
Form drag at TTP

Internal form drag

- High pressure
- Low pressure

Frictional drag: $2 \times 10^6$ N
Form drag: $20-50 \times 10^6$ N

External form drag

- High pressure
- Low pressure

Max form drag: $200 \times 10^6$ N

Edwards et al., 2004

McCabe et al., 2006
Cruise at TTP in Dec. 2009

Puget Sound, WA

Pressure sensors (PPODs) at Three Tree Point (TTP)
PPOD pressure signals

Raw PPOD data [psi]

[Graph showing pressure data over time and depth, with pressure signals indicated by lines and markers.]

12/03  12/05  12/07  12/09

-200 -100  0    100
-122.41 -122.39 -122.37 -122.35

-122.41 -122.39 -122.37 -122.35

47.48  47.47  47.46  47.45  47.44  47.43  47.42

-200 -100  0    100

[Color bar indicating depth in meters (m).]
Oscillatory dynamics

- Converted tidal energy to internal waves, eddies, and mixing
- No tidally averaged work done on system
- Tidal energy converted to internal waves, eddies, and mixing

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**Graphs:**
- **Velocity (m/s):** Flood and slack tides
- **Form drag:** N (m/s^2)
- **Power:** W (W)
- **Cumulative power:** W (W)

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**Equations:**
- Total form drag:
- Drag due to tilt
- Residual drag

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**References:**
Warner and MacCready, 2009

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**Diagram:**
- Flood & ebb tides: waves and eddies
- Slack tide: background tilt

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Warner and MacCready, 2009
Background sea surface slope

- Tidal current [m/s]
- Tidal height [m]
- Sea surface slope [mm/km]

Background sea surface slope from PPODs

Estimated background sea surface slope based on a tidal prediction model of Puget Sound (in quadrature with velocity)

Sea surface slope from PPODs
Calculating the residual pressure

1. PPOD data with means removed

2. minus average tidal signal

3. minus background tilt

4. Residual pressure signals
Residual pressure signal

Offset plot of pressure [m of SSH]
Residual drag

Residual pressures

Residual form drag

\[ \text{form drag per unit width} = \int p \frac{\partial h}{\partial x} \, dx \]

\[ \text{wave drag per unit width} = \rho_0 U h_{max}^2 \]
Residual drag

\[
\text{form drag} \quad \frac{\text{unit width}}{\text{unit width}} = \int p \frac{\partial h}{\partial x} \, dx
\]

\[
\text{tidal velocity } (U)
\]

\[
\text{conversion rate} \quad \frac{\text{unit width}}{\text{unit width}} = \frac{\text{form drag}}{\text{unit width}} \times U
\]

Running mean: \(\sim 300\) W/m

\(\rightarrow\) times width: \(\sim 0.7\) MW
ROMS model of TTP

Modeling the Salish Sea
David Sutherland

http://faculty.washington.edu/dsuth/MoSSea/
ROMS model of TTP

surface vorticity [1/s]
Conclusions / future work

- Bottom pressure sensors can be used to measure form drag on topography in the ocean.

- A numerical model will help to quantify the total form drag and understand the physical processes that play a role in energy conversion at Three Tree Point.

- Parameterizations of form drag in the ocean will eventually be possible.