Abstract

This paper develops a model with overlapping generations where the household’s optimal fertility, child labor, and education decisions depend on the parent’s expectations or beliefs. Specifically, it is shown that there exists a range of parental income where the fertility rate is high, the children participate in the labor market and receive an incomplete education if a parent believes the return to education is low. The fact that the children participate in the labor market reduces their ability to accumulate human capital as a result of a negative child labor externality. Thus, the action of sending the children into the labor market is sufficient to ensure that the parent’s initially pessimistic expectations are fulfilled. On the other hand, if the parent believes the return to education is high, then fertility rate is low, and each child receives a complete education (no child labor). This action, in turn, fulfills the household’s optimistic beliefs since the children do not incur the negative child labor externality. It is then shown that a one time policy intervention, such as a banning of child labor and mandatory education, can be enough to move a country from the positive child labor equilibrium to the no child labor equilibrium by temporarily removing the high fertility/child labor/incomplete education equilibrium from the household’s choice set when parental income falls within this ‘expectations’ range. Furthermore, it is also shown that this type of policy intervention either reduces household welfare if parental income is below this ‘expectations’ range or is unnecessary above it. Thus, policy effectiveness depends on the stage of the development process.

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Expectations, Child Labor and Economic Development

Poverty and child labor are inexorably linked: poor households are often forced to make difficult decisions about current consumption and future income when deciding the number of children to have, the amounts of educational inputs for their children and how much to have them work. In making such decisions families are required to forecast the future returns to education. The actual returns to education, however, will likely depend on a number of factors, including the growth of the overall economy and inputs into the education infrastructure by the government. It is unrealistic to expect perfect foresight on the part of poor households, so these households will likely extrapolate the future returns to education based on their own experiences. It is also likely that working as a child will harm the overall human capital attainment of individuals, so the adult labor market experiences of parents who were child laborers may be quite different then those who were not.¹

This paper presents a model with overlapping generations that attempts to capture these features. To accomplish this we make the following stylized assumptions. First, we assume that child labor reduces a child’s ability to accumulate human capital.² This implies that children who participate in the labor market receive a lower return on their education investment. Second, we assume that parents do not internalize this individual specific negative child labor externality. That is, parents are boundedly rational in the sense that they do not fully understand the complex relationship between child labor and the child’s developmental process.

¹ There is strong empirical evidence to support this claim. See Emerson and Souza (2007).
² This could be the result of, for example, reducing study time, poor health, fatigue, stress or anything else associated with child labor that hinders a child’s ability to learn.
Within this framework we demonstrate the existence of three mutually exclusive ranges of parental human capital. In the first range, a poverty range, parents decide to have relatively more children, send their children into the labor market, and provide them with an incomplete education. It is shown that this decision is independent of the return to education; thus, the household’s decision is unique. In the second range, a prosperity range, parents decide to have fewer children, provide their children with a complete education, and decide not to send them into the labor market. Once again, this decision is independent of the return to education and is unique. In the third range, an expectations range, we demonstrate that the parent’s beliefs interact with the parent’s human capital, to select one of the equilibrium paths described above. In other words, within this expectations range the household’s decision is no longer governed solely by the economy’s fundamentals.

The intuition is straightforward. For levels of parental human capital within the poverty range the parent’s earnings are so low that it is optimal to have more children and to send these children into the labor market to provide for household consumption. At the other end of the spectrum, when the level of parental human capital falls within the prosperity range, the parent’s earnings are sufficient to provide for household consumption and it is optimal to have fewer children and to provide these children with a complete education. These ranges match closely with other studies of child labor dynamics and with the luxury axiom of Basu and Van (1998) which states that families will only send children to work if forced to do so by economic necessity.

Within the expectations range, however, the luxury axiom does not necessarily hold and initial conditions are insufficient to determine the household’s optimal choice.
Specifically, within this range, the equilibrium selection mechanism depends on the state of the economy and the household’s beliefs in a way that fulfills the household’s expectations. That is, if a parent believes the return to education is high, then this parent will have fewer children and each child’s education will be complete (no child labor), which fulfills the household’s initially high expectations. If this parent believes the return to education is low, then this parent will have more children and send these children into the labor market. The fact that the children participate in the labor market implies that they incur the negative child labor externality, which reduces their ability to accumulate human capital. Thus, because the parent believes the return to education is low, the parent undertakes actions that fulfill this initially pessimistic expectation. Thus, for apparently ex-ante identical households it would appear ex-post that they came from completely different demographic and economic regimes.

As a final exercise we demonstrate that the existence of an expectations range introduces an important role for government policy. In particular, as argued by Evans and Honkapohja (1993), we show that government policy can be used to steer expectations away from a ‘bad’ equilibrium. For example, banning child labor and mandating education will force households to learn that the return to education is higher than they believed. Thus, by removing the child labor, incomplete education, high fertility option from the household’s choice set, the household is forced to internalize the negative effects of child labor and move to the Pareto superior outcome with low fertility, complete education, and no child labor. Once the household’s expectations have been altered there will no longer be any need to enforce the child labor and education laws because families will now internalize this high return to education and choose to educate
their children in the future. This ‘benign’ policy intervention will also reduce fertility as households substitute child quality for quantity.\(^3\) It is also shown that if a government implements a policy that bans child labor and mandates education within the ‘poverty range’, this will unambiguously reduce household welfare for the current households. Thus, the appropriateness and effectiveness of a ban on child labor may not only depend on the cause of child labor, but the stage of the development process.

The idea that expectations can serve as an equilibrium selection mechanism has a long history in development, dating back to the seminal work of Rosenstein-Rodan (1943) and the theory of the “big push”. Murphy, Schleifer and Vishny (1989) formalize this argument and show that expectations over market size can determine the equilibrium outcome.\(^4\) In addition, Dessy and Vencatachellum (2003) and Dessy and Pallage (2001) show that expectations or beliefs may select a child labor/no education equilibrium in the presence of strategic complementarities and human capital spillovers. That is, if all households choose a complete education then the return to education is high enough to support this equilibrium. On the other hand, if some households deviate from this choice then the return to education will be too low to support this equilibrium and we will observe complete child labor. Thus, in these two frameworks policy serves as a

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\(^3\) A ‘benign’ policy is a one-time effort that moves an economy out of one of equilibrium and into another, thus requiring no further intervention (Basu, 2003). This argument is also consistent with the work of Dessy (2000), where child labor results from initial poverty, or history, rather than expectations.

\(^4\) Macroeconomic models of business cycles and price fluctuations, studied in Cole and Kehoe (2000), Farmer and Woodford (1997), Farmer and Guo (1994), Evans and Honkapohja (1993) and Azariadis, (1981), to cite but a few, also discuss how expectations can serve as an equilibrium selection mechanism. Krugman (1991) provides a discussion about the history versus expectations debate. Additionally, there are macro search models (e.g. Diamond and Fudenberg, 1989), network externalities in industrial organization (e.g. Farrell and Saloner, 1986), and a well-established micro literature building on the seminal work of Cass and Shell (1983), where expectations or beliefs matter.
coordination mechanism, whereas, in our framework policy serves as a learning mechanism.\(^5\)

Our analysis is also related to the work of Basu and Van (1998), which highlighted the potential role of multiple equilibria in the context of child labor, and the work of Dessy and Pallage (2005), Dessy and Vancatachellum (2003), Dessy (2000), Becker, Murphy, and Tamura (1990), and Azariadis and Drazen (1990) who demonstrate that initial levels of human capital can influence the development path of a particular country. The results in these papers typically rely on nonlinearities in the state space alone to generate the interesting dynamic properties we observe across countries and households, whereas our paper highlights the potential role of expectations in the presence of these nonlinearities when agents are boundedly rational.\(^6\)

Finally, our paper relates to a number of other recent studies of child labor which incorporate the fertility decision. Doepke and Zilibotti (2005) demonstrate that political economy factors may influence the household’s child labor and education decisions and may also explain the decline in fertility during this transition via a household size lock-in effect.\(^7\) Moav (2005) suggests that this pattern of rising education, declining child labor, and declining fertility is the result of child quality costs falling relative to the quantity costs as parent’s become more educated. Doepke (2004) suggests that this pattern of development is the result of different technologies at different stages of the development


\(^6\) There is an additional distinction between the two types of models cited above. First, there is the type where multiple equilibria exist for a given initial condition (for example, Basu and Van’s (1998) labor market model). Second there is the type where different initial conditions result in different dynamic trajectories (for example, Becker, Murphy, and Tamura (1990) and Azariadis and Drazen (1990). As we will demonstrate, our model encompasses both types of equilibria.

\(^7\) Krueger and Tjornhom (2001) also provide a theoretical framework that attempts to explain the historical evolution of child labor laws.
process and different education policies across countries. Hazan and Berdugo (2002) make a similar argument that technological progress (access) may also result in multiple equilibria, one with child labor and the other without child labor. Our analysis differs from these endogenous fertility models by highlighting the potential role that a negative child labor externality, in conjunction with bounded rationality and expectations, may play in the development process.

The paper proceeds as follows. In the next section, the stylized economy is presented illustrating the role of expectations and the negative health externality. The dynamic behavior of child labor and education are described in section three. The fertility decision is described in section four. The long-run growth and policy implications of the model are discussed in section five. Some modeling issues are discussed in section six. A summary and conclusion is presented in section seven.

II. The Stylized Economy

As mentioned, our theoretical analysis is based on two key assumptions. One, that child labor reduces a child’s ability to accumulate human capital. Two, that the parent of the household is boundedly rational. That is, the parent does not directly internalize this negative human capital effect. However, before these properties are incorporated into the model, the representative household’s general decision problem is described.

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8 This historical argument is also prevalent in the literature attempting to explain the long-term development process and demographic transition (see, e.g., Hansen and Prescott, 2002; Galor and Weil, 2000; and Goodfriend and McDermott, 1995).

9 There are two additional strains of child labor literature. There is the ‘fundamentals’ argument made by Emerson and Knabb (2006), which suggests that child labor is the result of other policies or economic circumstance, and the credit constraint argument put forth by Baland and Robinson (2000) and Ranjan (2001).
A. The Household’s Problem

Consider a stylized economy with overlapping generations that live for two periods. At the beginning of each period a household unit is composed of a single adult born in period $t - 1$. The adult brings $h_t > 0$ units of human capital or parental income into the current period (these two terms are interchangeable in the model), decides how many children to have, $n_t > 0$, and how each child allocates a unit of time between education, $e_t \in [0,1]$ and participation in the labor market, $(1 - e_t) \in [0,1]$. For simplicity, it is assumed that each child is treated symmetrically within the family.\(^{10}\)

For expositional purposes, and to maintain analytical tractability, we use specific functional forms in our analysis. This will allow us to highlight the potential role of parental beliefs when agents are boundedly rational in the development process. Specifically, we assume that parental utility is a function of household consumption $c_t$ and the aggregate wealth of the household’s children $n_t h_{t+1}$ (paternalistic altruism):\(^{11}\)

$$u(c_t, n_t h_{t+1}) = \ln(c_t) + \beta(h_t) \ln(n_t h_{t+1}).$$

The additional parameter $\beta(h_t) \in [0,1]$ measures the degree of parental altruism towards their offspring. Specifically, it is assumed that wealthier households assign less weight to their children’s total future earnings, $\beta'(h_t) < 0$. This could be the result of selfish behavior or because the parent understands that each child will be relatively more

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\(^{10}\) There are a number of papers that explicitly study the effects of birth order on allocations, outcomes and child labor in developing countries. For example, see Emerson and Souza (2002), Ejrnæs and Pörtner (2004), Behrman and Taubman (1986) and Horton (1988).

\(^{11}\) This form of paternalistic altruism is consistent with the modeling strategies of Dessy and Pallage (2005), Moav (2005), Dessy and Vencatchellum (2003) Hazan and Berdugo (2002), and Galor and Weil (2000), to cite a few. The alternative is to assume that the household’s preferences are of the non-paternalistic form described by Barro and Becker (1989). This latter modeling strategy is employed by Emerson and Knabb (2006) in the context of child labor. Although, there is some empirical evidence that suggests the paternalistic form of altruism appears consistent with the data (see, Altonji, et.al., 1996), we provide a heuristic argument in section VI that suggests our results are robust to the specific form of altruism.
prosperous in the future, thus, more capable of taking care of themselves as adults. However, we note here that most of the results do not depend on this particular modeling device. In particular, this relationship provides the necessary income and substitution effects that allow for balanced growth, a constant fertility rate, when the economy is the prosperity range (discussed shortly). We demonstrate in section VI that all of the other results in the paper are qualitatively the same under the more standard assumption of a constant generational discount rate, \( \beta \in (0,1) \).

The household also faces the following constraints. First, consumption depends on the adult’s income, the number of children in the household, the amount of time each child spends in the labor market, and a child rearing cost parameter \( \nu \in (0,1) \).

\[ (2) \quad c_t = (1 - \nu n_t) h_t + (1 - e_t) n_t. \]

Implicit in this constraint is a linear technology that combines the labor of each child, \((1 - e_t) n_t\), and the adult’s human capital in conjunction with the adult’s time allocated to the labor market, \((1 - \nu n_t) h_t\), to produce the final consumption good.\(^\text{12}\)

The second constraint is the education technology. This function maps the time a child spends receiving an education to his or her respective level of adult human capital.

\[ (3) \quad h_{t+1} = f(e_t) h_t, \quad \text{where} \quad f'(e_t) > 0, \quad f(0) = 1, \quad \text{and} \quad f(1) = \theta > 1 \]

By assumption, a child’s human capital, supplied as an adult, is an increasing function of the time spent receiving an education and parental human capital. The property \( f(0) = 1 \) imposes a ‘no growth’ condition on the economy when child labor is complete and the

\(^{12}\) For other uses of this specification, see Hansson and Stuart (1989), Glomm and Ravikumar (1992), and Baland and Robinson (2000).
property $f(1) = \theta > 1$ imposes a ‘positive growth’ condition on the economy when education is complete.

**B. The Effect of Child Labor on Human Capital**

As previously noted, it is assumed that if a child works, then this will adversely affect the child’s ability to accumulate human capital. We represent this potential negative child labor externality with the following function:

$$\alpha_t = \begin{cases} \lambda & \text{iff } e_t = 1 \\ \rho(e_t)\lambda & \text{iff } e_t \in [0,1) \end{cases}$$

Where $\alpha_t$ is the elasticity of human capital with respect to the time a child spends receiving an education, or formally,

$$\alpha_t = \frac{f''(e_t)e_t}{f(e_t)} \in [0,1).$$

These two equations state that if a child’s education is complete (no child labor) then this elasticity, or what we refer to as the *quasi return to education*, equals its postulated constant maximum value, $\lambda \in (0,1)$. If, however, child labor is present (an incomplete education), then the return to education is potentially below its maximum value as a result of the negative child labor externality. We measure the severity of this externality with the function, $\rho(e_t) \in [0,1]$, where $\rho'(e_t) \geq 0$. This implies that the child’s ability to accumulate human capital is a weakly decreasing function of child labor.

There is a fair amount of empirical evidence consistent with this hypothesis. For example, Heady (2000) finds that a child’s participation in the labor market reduces his or her level of academic achievement. Alderman *et. al.* (2006) show that improvements in a child’s preschool health and nutrition result in an increase in school attendance. If we
extend this logic to the early childhood/pre-teen years, this suggests that there is a positive correlation between the child’s health and the time the child spends receiving an education. Emerson and Souza (2007) find that child labor has detrimental effects on adult labor market outcomes net of the quantity of education, suggesting detrimental human capital consequences from child labor. At the theoretical level, this line of reasoning is consistent with the seminal paper by Becker, et. al., (1990) who suggest that the return to education increases directly with the time a child spends receiving an education. It is also consistent with a recent paper by Dessy and Pallage (2005) which argues that the worst forms of child labor reduce a child’s ability to accumulate human capital.\textsuperscript{13}

The last component of the model is the forecasting rule. But, before we introduce this part of the decision process we first introduce the following condition.

**Assumption 2.1:** Let \( \alpha_t = \rho \lambda \) when \( e_t \in [0,1] \), which implies \( \rho'(e_t) = 0 \).

This assumption implies that the return to education, defined by equation (4), is a standard step (threshold) function. This initial setup allows us to highlight the potential role of beliefs in an economy with boundedly rational agents. In section VI of the paper we argue that most of the results are robust to this simplifying assumption.

With this simplification in hand, the household’s forecasting rule is as follows.

\textsuperscript{13} An alternative to this assumption is that the return to education is subject to peer effects or strategic complementarities, as in Dessy and Venca
tachellum (2003). Thus, the return to education depends on the decisions made by other households. Our setup depends on individual decisions and bounded rationality.
Forecasting Rule (Assumption 2.2): If a parent realizes a low return on his or her education investment as an adult, then the forecast (belief) is that each child will also realize a low return during their adult years, $\alpha_i = \rho \lambda$. On the other hand, if a parent realizes a high return on his or her education investment as an adult, then the forecast (belief) is that each child will also realize a high return during their adult years, $\alpha_i = \lambda$.

This rule states that when forecasting the return to education for their children, the parents use their own experience as a guide.\(^{14}\) It will be shown that this forecasting rule, in conjunction with the negative child labor externality, can result in a virtuous or vicious circle over a certain range of the development process.\(^ {15}\)

C. The Dynamical System

It is straightforward to show that the combination of the household’s objective function, given in equation (1), the household’s budget constraint, given in equation (2), the education technology, given in equation (3), and the return to education (elasticity), given in equation (4), result in the following optimality conditions for $\{c_i, e_i, n_i\}$, conditional on the state variable $h_i$ (see appendix).

\begin{align*}
(2) & \quad c_i = (1 - vn_i)h_i + (1 - e_i)n_i \\
(3) & \quad h_{i+1} = f(e_i)h_i
\end{align*}

\(^{14}\) Perhaps the best way to conceptualize expectations in the current model is to assume that all households have the same set of beliefs. In fact, we will demonstrate that this is indeed the case in section five. Although it is also important to note that our results do not directly depend on this assumption. Our setup does not require coordinating beliefs between agents because our externality does not depend on agglomeration externalities or strategic complementarities. Our results depend only on the household’s own boundedly rational decisions.

\(^ {15}\) Dessy and Pallage (2005) demonstrate that the household may choose to send their children into the labor market even if they do understand or internalize the negative human capital effects of child labor. Their result follows from an endogenous wage premium that is paid to ‘risky’ forms of child labor and its interaction with poverty. We demonstrate shortly, that there exists a range of parental income where expectations or beliefs play a significant role in the development process once we relax this assumption.
Equations (2) and (3) have already been defined. Equation (6) determines the amount of time each child spends receiving an education, or alternatively in the labor market. The first entry describes an interior decision and also captures the lower bound case, \( e_i = 0 \), given the parameterization of the economy. The second entry describes the upper bound education decision (complete education and no child labor). The properties of this decision rule are summarized with the following lemma.

**Lemma 2.1:** (All proofs have been relegated to the appendix.)

1. If \( \alpha_i \in (0,1) \) and \( e_i \in (0,1) \) then the time a child allocates to education is an increasing function of parental income, \( \frac{\partial e_i}{\partial h_i} > 0 \), which implies that child labor is a decreasing function of parental income, \( \frac{\partial (1 - e_i)}{\partial h_i} < 0 \).

2. If \( \alpha_i \in (0,1) \) and \( e_i \in (0,1) \) then the time a child allocates to education is an increasing function of the return to education, \( \frac{\partial e_i}{\partial \alpha_i} > 0 \), which implies that child labor is a decreasing function of the return to education, \( \frac{\partial (1 - e_i)}{\partial \alpha_i} < 0 \).

3. If the externality is complete, \( \rho = 0 \), which implies that \( \alpha_i = 0 \) in the presence of child labor, then the household’s choice is binary \( e_i = \{0,1\} \) (Discrete Choice Model).
Equation (7) determines the number of children born into each household.\textsuperscript{16} This decision depends on properties of the model that have yet to be formally addressed. Thus, the discussion of the fertility decision will be postponed until section IV.

**III. Education and Child Labor**

With the household’s optimal decision rules in place, and an understanding of how child labor can affect the child’s ability to accumulate human capital, we now formally describe the household’s education/child labor decision and demonstrate how this decision depends on the level of parental human capital.

**A. History and Beliefs**

For the sake of interest, a lower bound is placed on the initial level of human capital. Specifically, let \( h_i > 1/\nu \). This assumption ensures that the household’s education decision is strictly positive and that child labor does not encompass all of the child’s time, as long as the return to education is strictly positive.\textsuperscript{17} A condition we now temporarily impose on the economy.

**Assumption 3.1:** Let \( \rho \in (0,1] \), which implies that \( \alpha_i > 0 \) for all \( e_i \in (0,1] \).

The case where the child labor externality is complete will be addressed separately.

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\textsuperscript{16} Note that since we do not include child mortality in our model the total fertility and net fertility rates are equivalent. See Hazan and Zoabi (2006) for more on this issue.

\textsuperscript{17} If we were to relax this assumption and let \( h_i \leq 1/\nu \), then we would introduce another form of poverty trap, an extreme poverty trap. We instead choose to highlight the poverty trap generated by a complete negative externality, \( \rho = 0 \).
The first exercise is to define a lower range, or ‘poverty range’, of human capital where child labor is present and independent of the return to education. Consider the following proposition.

**Proposition 3.1:** There exists a poverty range of parental income \( h_i \in (1/v, 1/\lambda v) \) such that the household optimally chooses to send their children into the labor market and provide them with an incomplete education independent of the perceived return to education, \( \alpha_i \in \{\rho \lambda, \lambda\} \).

One can easily verify this result by substituting the two possible returns to education, \( \alpha_i = \lambda \) and \( \alpha_i = \rho \lambda \), into the education/child labor decision rule and showing \( e_i \in (0,1) \) for both (formal proof in appendix). Here households are too poor to afford a complete education and choose to send their children into the labor market, independent of initial beliefs. In addition, since each child participates in the labor market the realized return to education is low, \( \alpha_i = \rho \lambda \), within this income range. This last result implies that history alone selects a unique fulfilled expectations equilibrium with child labor and an incomplete education.18

Our second exercise is to define an upper range, or ‘prosperity range’, of human capital. In this case education is complete and independent of the return to education.

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18 There is a bit terminology abuse here on our part. We have not formally defined an equilibrium since the optimal fertility decision has not yet been derived or defined. But, given the closed form nature of the model, we will continue to use this terminology and provide the complete equilibrium dynamics in section IV. The closed form solution also demonstrates existence in the current context.
**Proposition 3.2:** There exists a prosperity range of parental income \( h \in [1/\rho \lambda \nu, \infty) \) such that the household optimally chooses to provide their children with a complete education and decides not to send their children into the labor market independent of the perceived return to education, \( \alpha_\nu \in \{\rho \lambda, \lambda\} \).

This result is also easily verified by the substitution method. Within this income range the household is wealthy enough to send their children to school full-time as long as the return to education is strictly positive, which holds under assumption 3.1. Again, since the realized return to education is high \( (\alpha_\nu = \lambda) \), history alone selects a *unique fulfilled expectations equilibrium* with no child labor and a complete education.

Note that these two propositions are consistent with Basu and Van’s (1998) luxury axiom. A relatively poor household (a household that falls within the poverty range) will choose to send their children into the labor market to provide for household consumption. On the other hand, a relatively wealthy household (a household that falls within the prosperity range) will choose to provide their children with a complete education. Crucially, however, there exists a gap between these two income ranges as long as the child labor externality is present, \( \rho \in (0,1) \). This begs the question, what happens within this middle income range?

The following proposition provides one potential answer.
Proposition 3.3: There exists an expectations range of parental income \( h_i \in \left[ \frac{1}{\lambda \nu}, \frac{1}{\rho \lambda \nu} \right] \) such that:

1. The household optimally chooses to send their children into the labor market and provides them with an incomplete education if they believe the return to education is low, \( \alpha_i = \rho \lambda \).

2. The household optimally chooses to provide their children with a complete education and decides not to send them into the labor market if they believe the return to education is high, \( \alpha_i = \lambda \).

This proposition implies that if a household believes the return to education is low, then each child will participate in the labor market. Since these children participate in the labor market they suffer from the negative human capital effects associated with this action. Thus, the realized return on the education investment is in fact low. On the other hand, if a household believes the return to education is high, they invest in each child’s education and choose not send them into the labor market. The fact that these children do not participate in the labor market and receive a complete education fulfills the household’s initially high expectations. This implies that it is the interaction of history and expectations (or beliefs) that determines the (boundedly) rational and optimal education/child labor decisions. Thus, we have multiple fulfilled expectations equilibrium paths for education and child labor.\(^{19}\)

We summarize this optimal education/child labor decision with the following functional equivalent:

\(^{19}\) Again, apply the substitution method for each possible belief within this income range and observe the education choice.
Note that the expectations range is bounded from below by the poverty (history) range and it is bounded from above by the prosperity (history) range when $\rho \in (0,1)$.

B. The Complete Child Labor Case and the Depth of Expectations Driven Poverty

From equation (8) we can now deduce the effect of the negative child labor externality when it is complete.

**Corollary 3.1:** If the negative child labor externality is complete, $\rho = 0$, and the child participates in the labor market, then the return to education is zero, $\alpha_i = 0$. This implies that the expectations range encompasses $h_i \in [1/\lambda v, \infty)$ and the prosperity range is degenerate. Furthermore, if parental income lies within the poverty range, or parental income lies within the expectations range and the household believes the return to education is low (zero), then child labor is complete and there is no investment in education, $e_i = 0$. Therefore, the education/child labor decision is binary.

In this scenario the potential role of expectations is no longer bounded from above. This implies that a zero education or complete child labor equilibrium always exists.
This proposition, in conjunction with the results derived in the previous section, allows us to demonstrate how the education/child labor decision depends on the severity of the negative child labor externality once the economy initially enters the expectations range, \( h_i = (1/\lambda \nu) \). Specifically, in Figure 1, the horizontal axis measures the severity of the externality, \( \rho \in [0,1] \). The vertical axis measures the time a child spends in the labor market \((1 - e_i) \in [0,1]\) and the time the child spends receiving an education, \( e_i \in [0,1]\). From this figure we can conclude that as the severity of the externality increases, we will observe an increase in child labor and a decrease in education for those households who believe the return to education is low upon initial entry into the expectations range.

IV. The Household Fertility Decision

The household fertility decision depends on two different factors. First, the optimality condition:

\[
\begin{align*}
(7) \quad n_i &= \alpha_i \beta(h_i) \left( \frac{c_i}{e_i} \right) \\

\end{align*}
\]

Second, whether the education decision is interior, \( e_i \in (0,1) \), or at the upper bound, \( e_i = 1 \). We address each of these cases in turn.

To determine the optimal number of children when the education decision is interior, and the parent believes the return to education is low, first eliminate consumption, \( c_i = (1 - \alpha n_i) h_i + (1 - e_i) n_i \), and education, \( e_i = \rho \lambda / (1 - \rho \lambda) (\nu h_i - 1) \), from the optimality condition. Then after some algebra, the following decision rule results:

\[
(9) \quad n_i = \frac{\beta(h_i)(1 - \rho \lambda) h_i}{(1 + \beta(h_i))(\nu h_i - 1)}
\]
If on the other hand the education decision is at the upper bound, \( e = 1 \), and the parent believes the return to education is high, then the optimality condition becomes \( n_t = \lambda \beta(h_t) c_t \) and consumption becomes \( c_t = (1 - vn_t)h_t \). After substitution, and a little algebra, we have the following decision rule:

\[
(10) \quad n_t = \frac{\lambda \beta(h_t) h_t}{1 + \lambda v \beta(h_t) h_t}.
\]

These fertility equations can be further simplified with the following lemmas.

**Lemma 4.1:** Let \( \beta(h_t) = 1/(Kh_t) \), which satisfies the condition \( \beta'(h_t) < 0 \). This implies:

1. If \( e_t \in (0,1) \) then the fertility decision becomes:

\[
(11) \quad n_t = \frac{(1 - \rho \lambda) h_t}{(1 + Kh_t)(vh_t - 1)}, \quad \text{where} \quad \frac{dn_t}{dh_t} < 0.
\]

2. If \( e_t = 1 \) then the fertility decision becomes:

\[
(12) \quad n_t = \frac{\lambda}{K + \lambda v}, \quad \text{where} \quad \frac{dn_t}{dh_t} = 0.
\]

Two important results are achieved by imposing this functional representation on the model. First, as parental income increases, the fertility rate declines when child labor is present. Thus, the substitution effect dominates the income effect. This result is consistent with the empirical evidence provided by Rosenzweig (1990), who finds that fertility does in fact decrease when income and education increase (or child labor decreases), thus capturing the standard Barro-Becker (1989) child quality-quantity
tradeoff. Second, when education is complete, the fertility rate is now constant, 
\[ g_n = n - 1 = \left[ \lambda(1 - v) - K \right] \left[ K + \lambda v \right], \] 
which is consistent with balanced growth dynamics.

Now consider the following additional restriction.

**Lemma 4.2:** Assume that \( K = \lambda(1 - v) \). This implies:

1. If \( 0 < e_t < 1 \) then the fertility decision becomes:

   \[ n_t = \frac{(1 - \rho \lambda) h_t}{[1 + \lambda(1 - v) h_t(vh_t - 1)]}, \text{ where } \frac{dn_t}{dh_t} < 0. \]

2. If \( e_t = 1 \) then the fertility decision becomes:

   \[ n_t = 1, \text{ where } \frac{dn_t}{dh_t} = 0. \]

This additional restriction implies that the population is constant once child labor is eliminated from the economy, \( g_n = 0 \). If \( K < \lambda(1 - v) \) then the population growth rate would be positive, \( g_n > 0 \). If \( K > \lambda(1 - v) \) then the population growth rate would be negative, \( g_n < 0 \). Since neither of these two extensions would change the qualitative results, the stronger results in Lemma 4.2 are employed.

Given the properties in Lemma 4.2, the relationship between the household’s fertility decision, parental income, and beliefs are formally defined with the following proposition:
Proposition 4.1:

(1) There exists a poverty range of parental income \( h_i \in (1/v, 1/\lambda v) \) such that the household optimally chooses a relatively high fertility rate.

(2) There exists a prosperity range of parental income \( h_i \in [1/\rho \lambda v, \infty) \) such that the household optimally chooses a relatively low fertility rate.

(3) There exists an expectations range of parental income \( h_i \in [1/\lambda v, 1/\rho \lambda v) \) such that:

(A) The household optimally chooses a high fertility rate if the adult believes the return to education is low, \( \alpha_i = \rho \lambda \).

(B) The household optimally chooses a low fertility rate if the adult believes the return to education is high, \( \alpha_i = \lambda \).

The key difference between this theory and its predecessors (Becker, Murphy, and Tamura, 1990; Galor and Weil, 2000; Dessy, 2000; and Hazan and Berdugo, 2002) is that when the parent’s human capital falls within the expectations range, \( h_i \in [1/\lambda v, 1/\rho \lambda v) \), the fertility decision will also depend on the parent’s beliefs, rather than income alone.

The underlying logic of this argument is straightforward. If a parent’s human capital is within the poverty range \( h_i \in (1/v, 1/\lambda v) \) then the education decision is interior and there is positive child labor. This induces the parents to have more children to send into the labor market. This decision is also independent of the return to education (Proposition 3.1). Thus, equation (13) determines the household’s fertility decision along a unique fulfilled expectations path. This result also holds if the parent’s human capital is
in the expectations range $h_i \in [1/\lambda \nu, 1/\rho \lambda \nu)$ and the parent believes the return to education is low (Proposition 3.3).

On the other hand, if the parent’s human capital is within the prosperity range $h_i \in [1/\rho \lambda \nu, \infty)$ then the education decision is at the upper bound and child labor is absent. This decision is independent of the return to education (Proposition 3.2). Thus, equation (14) determines the household’s fertility decision along a unique fulfilled expectations path. This result also holds if the parent’s human capital is in the expectations range $h_i \in [1/\lambda \nu, 1/\rho \lambda \nu)$ and the parent believes the return to education is high (Proposition 3.3).

We now summarize the households’ optimal fertility decision with the following functional equivalent.

\[
 n_i = \begin{cases} 
 \frac{(1 - \rho \lambda) h_i}{(1 + \lambda (1 - \nu) h_i)(\nu h_i - 1)} & \text{iff } h_i \in \left(1/\lambda \nu, 1/\lambda \nu\right) \\
 \frac{(1 - \rho \lambda) h_i}{(1 + \lambda (1 - \nu) h_i)(\nu h_i - 1)} & \text{iff } h_i \in \left[1/\lambda \nu, 1/\rho \lambda \nu\right) \text{ and belief } \alpha_i = \rho \lambda \\
 1 & \text{iff } h_i \in \left[1/\lambda \nu, 1/\rho \lambda \nu\right) \text{ and belief } \alpha_i = \lambda \\
 1 & \text{iff } h_i \in \left[1/\rho \lambda \nu, \infty\right)
\end{cases}
\]

Again, note that the expectations range is bounded from below by the poverty (history) range and it is bounded from above by the prosperity (history) range when $\rho \in (0,1)$.

As a final exercise, we discuss the case where the child labor externality is complete. Equation (15) shows that when parental human capital equals $h_i = [1/\lambda \nu]$ (initial entry) and child labor is present we have the following fertility choice.
As shown in Figure 2, if the human capital externality is absent, \( \rho = 1 \), then the fertility decision equals one, the expectations range vanishes, and child labor disappears. On the other hand, when the externality is complete, \( \rho = 0 \), and \( e_i = 0 \), then the fertility rate equals \( n_i = 1/[v(1 - \lambda)] > 1 \). For those values between these two extreme cases, \( \rho \in (0,1) \), we can see that the fertility rate increases with the severity of the externality.

V. Dynamic Properties of the Model and Welfare Issues

The propositions, corollaries, and lemmas in the previous sections demonstrated how the state variable parental human capital, or income, could be divided into three mutually exclusive ranges in the presence of a negative child labor externality when agents are boundedly rational. It was then shown that the state variable either selected a unique fulfilled expectations equilibrium, or interacted with expectations to select from multiple fulfilled expectations equilibrium paths. The focus now shifts to the dynamic implications of this model, assuming assumptions 2.1 and 3.1 hold.

A. The Pattern of Economic Development

We begin by assuming that the initial level of human capital falls within the poverty range, that is \( h_0 \in (1/v, 1/\lambda v) \), where \( t = 0 \) defines the starting date. This implies that the households within this initial generation will optimally choose to send their children into the labor market (Proposition 3.1) and will also decide to have relatively more children (Proposition 4.1). Furthermore, since each child incurs the externality they receive a low return on their respective education investments.
Our next exercise is to trace out the dynamic properties of this stylized economy as it moves through each respective stage of the development process. Given that the initial generation’s human capital falls within the poverty range and there is some investment in education, or child labor is incomplete (Assumption 3.1), we can conclude that human capital or parental income is increasing at an accelerating rate across generations. Thus, the state variable $h_t$ is an increasing sequence within this range. This result follows directly from the human capital accumulation equation, $h_{t+1}/h_t = f'(e_t) > 1$, and the following set of assumptions, $f(0) = 1$, $f'(e_t) > 0$, and $f(1) = \theta > 1$. In addition, we also know that as parental human capital increases, child labor decreases and education increases (Lemma 2.1). Finally, the fertility rate is also declining during this phase of the development process as households substitute child quality for quantity (Lemma 4.2).

We summarize these results with the following **Poverty Range Dynamics**.²⁰

\[(17a) \begin{equation*} h_{t+1}/h_t = f(e_t), \quad e_t = \left(1 - \frac{\rho \lambda}{1 - \rho \lambda}\right)(vh_t - 1), \quad n_t = \left(\frac{1 - \rho \lambda}{1 + \lambda(1 - v)h_t}(vh_t - 1)\right)_{t=0}^{T^*} \end{equation*} \]

Thus, there exists a *unique fulfilled expectations path* where education is incomplete and child labor persists within the poverty range of the development process.

Once the economy reaches the upper bound of the poverty range, $h_t = 1/\lambda v$, which occurs in finite time $T^*$, it enters the expectations phase of the development process. Here, there are two potential *expectations paths* the economy can follow.

---

²⁰ The following descriptions of the dynamic behavior of the household/economy, across income ranges, summarize our concept of equilibrium. This also demonstrates existence for the current stylized model. All households are maximizing household welfare given the constraints and the forecasting rule (beliefs).
(Propositions 3.3 and 4.1). First, the economy can remain in the same low growth regime with child labor, incomplete educations, and high fertility.

\[
(17b) \quad \left\{ \frac{h_{t+1}}{h_t} = f(e_t), \quad e_t = \left( \frac{\rho \lambda}{1 - \rho \lambda} \right)(\nu h_t - 1), \quad n_t = \frac{(1 - \rho \lambda)h_t}{(1 + \lambda(1 - \nu)h_t)(\nu h_t - 1)} \right\}^{T_e_{HU}}.
\]

Or, second, the economy can jump directly into the prosperity phase of the development process where child labor vanishes, education is complete, the fertility rate is at replacement, and growth is at its upper bound. Specifically, we observe the following

**Prosperity Range Dynamics** during the expectations phase of the development process if this jump is made.

\[
(18a) \quad \left\{ \frac{h_{t+1}}{h_t} = \theta, \quad e_t = 1, \quad n_t = 1 \right\}^{T_e_{HU}}.
\]

Which of these expectations paths will the economy actually follow during this phase of the development process? The answer depends on the parent’s beliefs or the expected rate of return on education. In our setting we have assumed that household’s form beliefs based on their own life experiences: If the return to education was low for the parent, then they believe the return will be low for their children. If the return to education was high for the parent, then they believe the return will be high for their children (Forecasting Rule, Assumption 2.2.)

Given this forecasting rule, in conjunction with the initial condition, this economy continues to follow the poverty range dynamics during the expectations phase of the development process (Sequence 17b). The logic behind this result is that the initial generation of parents entering the expectations range were, by construction, themselves child laborers. Thus, the parents believe the return to education is low and will optimally choose to send their children into the labor market and have relatively more children.
This cycle of poverty will perpetuate itself through the entire expectations range because each consecutive generation of children will spend time in the labor market and incur the negative child labor externality.

This cycle of poverty with child labor, incomplete education, and relatively high fertility will finally end once the economy reaches the lower bound of the prosperity range \( h_t = 1/\rho \lambda v \). This will also occur in finite time \( T_H \) (Assuming \( \rho \neq 0 \), Assumption 2.1). Once the economy enters this prosperity phase of the development process, child labor vanishes, education is complete, the fertility rate is at replacement, and growth is at its upper bound. The following system, once again, describes the Prosperity Range Dynamics.\(^{21}\)

\[
\begin{align*}
\left\{ h_{t+1}, e_t = 1, n_t = 1 \right\}_{t=1}^{\infty}.
\end{align*}
\]

We summarize this dynamic process using the timeline in Figure 3. First, for an economy that starts off within the poverty range we observe child labor, incomplete educations, relatively high fertility, and relatively slow economic growth (column 1). This pattern of development will continue through the expectations phase of the development process as a result of the household’s forecasting rule, which encompasses our assumption that agents are boundedly rational (Column 2-Row 1). This pattern of development will only change once the economy reaches the prosperity range. In this latter phase of the development process we observe an economy with no child labor, complete educations, replacement fertility, and relatively higher economic growth (Column 3).

\(^{21}\) Note: The difference between equations (18a) and (18b) are the time indices.
This dynamic process now raises the following important questions: Is there anything the government can do that will induce the economy to jump to the prosperity range sooner (Column2-Row2 of Figure 3)? And, if so, what are the welfare implications of this jump? We address each of these questions in turn.

B. Policy Intervention and Welfare Issues

The key to the dynamic analysis above is the imposition of an expectations rule (the forecasting rule) on the households and the provision of an initial condition such that child labor is present. This eliminates the indeterminacy within the expectations range. Without these additional restrictions nothing specific could be said about the dynamic behavior of the economy within this range because we have *multiple fulfilled expectations equilibrium* paths. That is, in general, we have the following **Expectations Range Dynamics:**

If \( \alpha = \rho \lambda \) (households believe the return is low), then,

\[
\begin{align*}
\frac{h_{t+1}}{h_t} &= f(e_t), \\
e_t &= \left( \frac{\rho \lambda}{1 - \rho \lambda} \right) (v h_t - 1), \\
n_t &= \frac{(1 - \rho \lambda) h_t}{(1 + \lambda (1 - v) h_t) (v h_t - 1)} \text{ for } T_L \leq T_H.
\end{align*}
\]

If \( \alpha = \lambda \) (households believe the return is high), then,

\[
\begin{align*}
\frac{h_{t+1}}{h_t} &= \theta, \\
e_t &= 1, \\
n_t &= 1 \text{ for } T_L \leq T_H.
\end{align*}
\]

Here, the timing of the transition can occur at an earlier date \( T_E \in [T_L, T_H] \), where the subscript ‘ \( E \) ’ denotes a change in expectations.

We now address our first question. What can the government do to move the economy onto the prosperity range equilibrium path? Suppose the government initially bans child labor and mandates education at time \( T_E \), which lies within the expectations range of the development process. This forces parents to pull their children out of the
labor market and provide them with complete educations. Thus, these children do not incur the negative child labor externality and they receive a high return on their respective education investments. This sets off an early cycle of prosperity by breaking the chain of child labor and low returns via the forecasting rule, making this a ‘benign’ policy. That is, once the policy is implemented and successful it is no longer needed. In other words, this policy forces the households to learn. If we assume that $T^*_e$, the time which the policy is implemented, is the time of initial entry into the expectations range this effect can be relatively large, as shown in Figures 1 and 2.

Therefore, the answer to our first question is yes. The government can move the economy onto the prosperity equilibrium path by removing the ‘child labor, incomplete education, high fertility, and low growth’ equilibrium from the household’s expectations range choice set. A more general argument can be found in Evans and Honkapohja (1993).

The answer to our second question, the welfare question, depends on the timing of the policy. As above, we initially assume that the economy is within the expectations range. Next, assume the government announces the policy before the fertility decision is made. This can be thought of as a phase-in of the policy (or a mandate on the number of children at replacement). In this case we know that the parents reduce fertility and increase education via the mandate(s). Given that the parents would have made this same decision if they believed (knew) the return to education was high, the initial household under this policy is no worse off via the axiom of revealed preference. However, all future generations would be strictly better off because the economy transitions to the prosperity range sooner. If the policy does not include a control on fertility (direct or
phase in) the initial generation may lose because the fertility decision is not optimal after the fact (this argument is similar to the one found in Doepke and Zilibotti, 2005). We do not address this additional complication in the current model.

Now consider the case where the government implements this policy before the economy reaches the expectations range. That is, assume that the households are still within the poverty range of the development process. If this is the case, we know that this policy will actually reduce household welfare. This result follows directly from the axiom of revealed preference. Even if the households believed (or knew) the return to education was high, the parents would still choose to send their children into the labor market.

In summary, these welfare results suggest that the effectiveness of a program that bans child labor and mandates education not only depends on the cause of child labor, but the stage of the development process. In other words, even if education opportunities are available and parents fail to internalize the negative effects of child labor, a ban on child labor will reduce the welfare of the poorest households (those that fall into the poverty range). Only when these households reach a critical range of income, the expectations range, does this policy improve welfare.

C. A Poverty Trap

As a final exercise, consider the case where the externality is complete. If human capital initially lies within the poverty range then we have the following dynamics.

\[ h_{t+1}/h_t = f(0) = 1, \quad e_t = 0, \quad n_t = \frac{h_0}{(1 + \lambda(1 - \nu)h_0)(vh_0 - 1)} \]

This system implies that child labor is complete and education is absent in the economy. As a result, human capital remains constant across generations and there is no growth.
The fertility rate also remains relatively high. Thus, we have the standard poverty trap scenario. The only way to move the economy out of this trap is to ban child labor and mandate education. But as previously shown, this policy will reduce household welfare. Thus, when a poverty trap is present we have a generational conflict over policy design and this economy may never escape without outside intervention.

On the other hand, if the initial level of human capital lies within the expectations range, which is no longer bounded from above, the system once again depends on the parent’s beliefs and is binary in nature.

If \( h_t \in \left[ \frac{1}{\lambda v}, \infty \right) \) and \( \alpha_t = \rho \lambda \) (low return), then,

\[
\left\{ \begin{array}{c}
h_{t+1}/h_t = f(0) = 1, \quad e_t = 0, \quad n_t = \frac{h_0}{(1 + \lambda(1-v)h_0)(vh_0-1)} \end{array} \right\}_{t=0}^\infty.
\]

If \( h_t \in \left[ \frac{1}{\lambda v}, \infty \right) \) and \( \alpha_t = \lambda \) (high return), then,

\[
\left\{ \begin{array}{c}
h_{t+1}/h_t = \theta, \quad e_t = 1, \quad n_t = 1 \end{array} \right\}_{t=0}^\infty.
\]

If the parents believe the return to education is low, they choose to send their children into the labor market. If the parents believe the return to education is high, they provide their children with complete educations. The difference here is that this can persist indefinitely into the future because the prosperity range of income is empty when the child labor externality is complete. Thus, the economy will never grow out of the child labor, no education, and high fertility equilibrium. In this case, a ban on child labor and mandatory education is necessary to move the economy onto the high growth path.
VI. Modeling Issues

Before concluding the paper, we discuss three important modeling assumptions. The first is the discount rate. If we employ the standard assumption that the discount rate is independent of parental income, $\beta(h_i) = \beta$, equations (9) and (10) become:

\[
(9b) \quad n_t = \frac{\beta(1 - \rho \lambda) h_t}{(1 + \beta)(\nu h_t - 1)}
\]

\[
(10b) \quad n_t = \frac{\lambda \beta h_t}{1 + \lambda \nu \beta h_t}
\]

From equation (9b) we can see that when child labor is present the number of children born is still inversely related to the parent’s human capital or income, $\partial n_t / \partial h_t < 0$. Thus, as before, the substitution effect dominates the income effect. The key difference here is that we now lose the balanced growth property with respect to fertility, equation (10b). That is, as parental human capital increases so does the number of children born. This implies that the income effect now dominates the substitution effect, rather than canceling out, within the prosperity range of the development process. Also note that equation (8) (the education choice) and the ranges describing the different stages of the development process do not depend on the parameter $\beta(h_i)$. Thus, the only result that rests on the assumption $\beta'(h_i) < 0$ is the balanced growth fertility dynamics in the absence of child labor. Based on the fact that the population growth rate is at or below replacement for most developed countries, we include this additional time component in the model.

The second assumption is that the form of parental altruism is paternalistic in nature. If we use the theoretical alternative, non-paternalistic altruism, highlighted in
Barro and Becker (1989), we conjecture our general results would carry through. We provide a heuristic argument here that lends support to this conjecture.

First, define household (dynastic) utility with the standard Bellman equation.

$$V(h_t) = \max\{U(c_t) + \beta(n_t) V(h_{t+1})\}$$

Subject to the same set of constraints:

$$c_t = (1-vn_t)h_t + (1-e_t)n_t \quad \text{(2)}$$

$$h_{t+1} = f(e_t)h_t \quad \text{(3)}$$

We also assume that $$\beta'(n_t) > 0$$ over some range. This is consistent with functional relationship $$\beta(n_t) = a(n_t)n_t$$ employed in the non-paternalistic altruism literature. The solution to this problem provides us with the following optimality condition. From the education choice we have:

$$e_t = \frac{\alpha_t \beta(n_t) h_{t+1} V'(h_{t+1})}{U'(c_{t+1}) n_t} \quad \text{(23)}$$

This equation shows that there is a positive partial relationship between the return to education and the time a child spends receiving an education. For example, suppose a parent believes the return to education is high and this belief is fulfilled, which implies,

$$e_t = \frac{\lambda \beta(n_t) h_{t+1} V'(h_{t+1})}{U'(c_{t+1}) n_t} \quad \text{(23b)}$$

If on the other hand the parent believes there is low return to education, then $$\alpha_t = \rho \lambda$$.

This results in the following decision.

$$e_t = \frac{\rho \lambda \beta(n_t) h_{t+1} V'(h_{t+1})}{U'(c_{t+1}) n_t} \quad \text{(23c)}$$

In the extreme case, when the externality is complete and $$\rho = 0$$, the education choice is obviously zero, $$e_t = 0$$, and child labor is complete, which once again fulfills
the parent’s beliefs. If we now increase \( \rho \in (0,1] \) above its lower bound, an incomplete externality, we can see that the education choice is still not complete and child labor remains under the appropriate curvature restrictions. The key here is that the return to education (elasticity) is multiplicative with respect to a marginal increase in the future value of the state variable \( h_{t+1} \). A similar argument applies to the fertility decision, again under appropriate curvature restrictions.

Thus it appears that the results in the current paper do not depend on the specific form of altruism employed in the household decision process. For this and other reasons we employ the more tractable form of altruism that allows us to highlight the dynamic role that a negative child labor externality can play in the development process when parents are boundedly rational. This form of paternalistic altruism also appears, at least to us, to be more consistent with the concept of bounded rationality. Finally, and perhaps the most compelling argument, this paternalistic form of altruism is consistent with a large part of the recent child labor literature, which allows for direct comparability.

The third assumption is that the severity of the externality is independent of the time a child spends in the labor market (Assumption 2.1). If we relax this assumption, the education decision is described by an implicit function in the presence of child labor.

\[
(8b) \quad e_t = \frac{\rho(e_t)\lambda}{1 - \rho(e_t)\lambda} (vh_t - 1)
\]

Given that \( \rho'(e_t) > 0 \), this implies that the return to education now depends not only on the child labor/education decision, but on the time allocated to each. This obviously complicates the dynamics, but we can easily conclude that if the parents believe the return to education is low based on their own experiences, and child labor is present, the
results from the previous sections still apply. The key difference here is that we lose the property that the parent’s expectations or beliefs are fulfilled within the expectations range, given that parental human capital is increasing in this stylized setting. Here, we will observe an adaptive learning process where each generation systematically underestimates the return to education.\textsuperscript{22} In addition, this adaptive learning process increases the rate at which child labor disappears from the economy, which is easily seen in equation (8b), and makes the expectations range endogenous, \( h_t \in [1/\lambda v, 1/\rho(\epsilon, \lambda v)] \).

Thus, we would expect to observe a more rapid evolution through the expectations range when the return to education depends on the time a child spends in the labor market.

\textbf{VII. Conclusion}

Human capital is vital to the economic growth and prosperity. Without an educated and skilled work force, sustained development is unlikely to occur. However, human capital takes costly investments in children and most of these costs are born by their families. The amount of these investments will be determined by, among other things, their expected payoff. If the expected payoff to education depends on the choices households make, then this can lead to child labor, incomplete educations, and high fertility. In other words, within the context of our framework, the mere perception of a low return to education can lower investment in human capital, limit economic growth, and lead to more child labor and increased fertility in a fulfilling manner.

This idea that expectations can serve as the equilibrium selection mechanism, instead of history or poverty alone, is also important from a policy standpoint. If expectations are in fact keeping an economy in a bad equilibrium with child labor,\textsuperscript{22} A more detailed learning model is beyond the scope of the current paper. Interesting extensions could examine how parents learn from each other in a world where agents are boundedly rational, or whether parents learn when there are siblings present.
governments could establish policies that create a disincentive to send children to work or an incentive to send children to school in order to move their countries from a ‘bad’ to a ‘good’ equilibrium by removing the bad equilibrium from the expectations choice set. These policies would not have to remain in place once the switch in equilibrium occurs, but serve as devices to steer the economy.

This study also emphasizes that child labor is not necessarily just a consequence of poverty but also a probable root cause of it. The mere perception by a family or adult that their children will be poor and will have little economic opportunity in the future is enough to ensure that their children will indeed be poor. Altering this perception or expectation is critical in order for a country to escape from poverty.
Appendix:
Overview of the Solution (system of equations)
The parent of the household maximizes the following household utility function:

\[
u(c_t, n, h_{t+1}) = \ln(c_t) + \beta(h_t)\ln(n, h_{t+1})
\]

Subject to the following constraints:

\[
c_t = (1 - vn_t)h_t + (1 - e_t)n_t
\]

\[
h_{t+1} = f(e_t)h_t
\]

Substitute (2) and (3) into the objective function, then optimize with respect to \(\{e_t, n_t\}\).

\[
(\alpha)\quad \frac{n_t}{c_t} = \frac{\beta(h_t)f^\prime(e_t)}{f(e_t)}
\]

\[
(\beta)\quad \frac{(\alpha c_t)(e_t)}{c_t} = \beta(h_t)
\]

Next, rearrange (A) into the following form:

\[
(\gamma)\quad \frac{n_t}{c_t} = \frac{(\beta(h_t))\left(f^\prime(e_t), e_t\right)}{c_t} = \frac{\alpha, \beta(h_t)}{e_t} \quad \text{or} \quad \frac{\beta(h_t)}{n_t} = \frac{e_t}{\alpha, c_t}
\]

Substitute the last part of (C) into equation (B):

\[
(\delta)\quad (vh_t - 1) + e_t = \frac{e_t}{\alpha, c_t} \Leftrightarrow (vh_t - 1) + e_t = e_t
\]

Solving for \(e_t\) in (D) provides us with equation (6) and further rearranging (C) provides us with equation (7). Thus, we have the following system of equations:

\[
(2)\quad c_t = (1 - vn_t)h_t + (1 - e_t)n_t \\
(3)\quad h_{t+1} = f(e_t)h_t \\
(6)\quad e_t = \min\left\{\frac{\alpha_t}{1 - \alpha_t}, \frac{(vh_t - 1), 1}{1 - \alpha_t}\right\} \\
(7)\quad n_t = \frac{\alpha_t}{\beta(h_t)} \left(\frac{c_t}{e_t}\right)
\]

Given the structure of the problem, the rest of the process can be derived using standard algebraic techniques and appropriate substitution as governed by the dynamical system. This is formally shown in the proofs below.

Proofs of Propositions, Corollaries, and Lemmas

Lemma 2.1: Proof: Follows directly from equation (5).

Proposition 3.1: Proof: To demonstrate the existence of this range assume \(e_t = 1\). This implies that \(\left(\frac{\lambda}{1 - \lambda}\right)(vh_t - 1) \geq 1 \Rightarrow h_t \geq \frac{1}{\lambda v}\), see equation (5) and (6). Thus, if \(h_t < \frac{1}{\lambda v}\), then equation (6) implies that \(e_t \in (0,1)\), which is a contradiction. Therefore, if \(h_t \in (1/v, 1/\lambda v)\), then the household’s education decision, \(e_t \in (0,1)\) and child labor decision \(1 - e_t \in (0,1)\), are independent of the return to education, \(\alpha_t \in \{\rho\lambda, \lambda\}\), and lies in the interior. From lemma 2.1 we know this is also holds true for the low return state, \(\alpha_t = \rho\lambda\) and all levels of human capital within the range \(h_t \in (1/v, 1/\lambda v)\).
Proposition 3.2: Proof: To demonstrate the existence of this range assume \( e_r \in (0,1) \). This implies that \( \left( \frac{\rho\lambda}{1 - \rho\lambda} \right) (vh_t - 1) < 1 \iff h_t < \frac{1}{\rho\lambda v} \), see equations (5) and (6). Thus, if \( h_t \geq \frac{1}{\rho\lambda v} \), then equation (6) implies that \( e_r = 1 \), which is a contradiction. Therefore if \( h_t \in \left[ \frac{1}{\rho\lambda v}, \infty \right) \) the household decision, \( e_r = 1 \) and \( 1 - e_r = 0 \), is independent of the return to education \( \alpha_r \in \{\rho\lambda, \lambda\} \). Also, from lemma 1 we know this is also true for the higher return state, \( \alpha_r = \lambda \) and all levels of human capital within the range \( h_t \in \left[ \frac{1}{\rho\lambda v}, \infty \right) \). ■

Proposition 3.3: Proof: We demonstrate this result in two parts. 1. First, assume the household believes the return to education is low, \( \alpha_r = \rho\lambda \), and has a level of human capital or parental income of, \( h_t = \frac{1 - e_r}{\rho\lambda v} \) for some arbitrarily small \( e > 0 \). If the household then chooses to send their children into the labor market based on this belief, equations (5) and (6) imply \( e_r = \left( \frac{1 - \rho\lambda - e}{1 - \rho\lambda} \right) < 1 \), which fulfills the household’s initial beliefs. Lemma 2.1 closes the argument since \( \partial e_r / \partial h_t > 0 \), which implies this result holds for all \( h_t \in \left[ \frac{1}{\rho\lambda v}, \frac{1}{\rho\lambda v} \right) \). 2. Second, assume the household believes the return to education is high, \( \alpha_r = \lambda \), and has a level of human capital or parental income of, \( h_t = \frac{1}{\lambda v} \). If the household then chooses to provide their children with a complete education based on this belief, equations (5) and (6) imply \( e_r = 1 \), which fulfills the household’s initial beliefs. Lemma 2.1 again closes the argument since \( \partial e_r / \partial h_t > 0 \), which implies this result holds for all \( h_t \in \left[ \frac{1}{\lambda v}, 1/\rho\lambda v \right) \). The combination of parts 1 and 2 demonstrates that over the range of human capital \( h_t \in \left[ \frac{1}{\lambda v}, 1/\rho\lambda v \right) \) beliefs and history determine the equilibrium choice. ■

Corollary 3.1: Proof: Follows directly from equation (8). ■

Lemma 4.1: Proof: This follows directly from substitution, equations (9) and (10), and then differentiating equations (11) and (12). ■

Lemma 4.2: Proof: This follows directly from substitution, equation (11) and (12) in Lemma 4.2. ■

Proposition 4.1: Proof: This proof follows directly from equation (15), given results from Lemma 4.1 and 4.2. ■
REFERENCES


Figure 1
The Depth of Expectations Driven Poverty and Child Labor upon Initial Entry into
the Expectation Range: $h_r = 1/\lambda v$

\[ e_t = \frac{\rho(1-\lambda)}{1-\rho \lambda} \]

\[ 1-e_t = \frac{(1-\rho)}{(1-\rho \lambda)} \]
Figure 2
The Depth of Expectations Driven Fertility upon Initial Entry into the Expectation Range: \( h_t = 1/\lambda v \)
Figure 3
Dynamic Flow Chart

History-Poverty Range
- Child Labor
- Incomplete Education
- Poor Health
- High Fertility, But Declining
- Slow Growth, But Accelerating

History-Expectations Range (Low Return)
- Child Labor
- Incomplete Education
- Poor Health
- High Fertility, But Declining
- Slow Growth, But Accelerating

History-Prosperity Range
- No Child Labor
- Complete Education
- Good Health
- Low Fertility
- High Growth

History-Expectations Range (High Return)
- No Child Labor
- Complete Education
- Good Health
- Low Fertility
- High Growth