Math 231  Counting  Activity 5

This activity is worth 20 points of course credit. See tentative calendar for due dates. Late activities are accepted at the discretion of your recitation instructor and a penalty may be imposed.

(1) (3 pts) Let $X = \{a, b, c, d, e\}$. How many strings of length 3 can be produced by using the elements of $X$ if repetition is allowed? What if repetition is not allowed? How many subsets of $X$ of size 3 are there?

(2) A basketball team has 5 guards, 3 centers, and 4 forwards.

(a) (1 pts) How many starting lineups of 5 players contain no centers?

(b) (1 pts) How many starting lineups of 5 players contain at least two forwards?

(c) (1 pts) How many starting lineups of 5 players contain at most 4 guards?
(3) Use the Binomial Theorem (given below) to find the following coefficients.

\[ \forall n \in \mathbb{N}, (x + y)^n = \sum_{k=0}^{n} C(n, k) x^{n-k} y^k. \]

(a) (1 pts) Find the coefficient of \( x^9 \) in \((1 - x)^{12}\).

(b) (1 pts) Find the coefficient of \( x^2y^7 \) in \((2x - y)^9\).

(c) (1 pts) Find the coefficient of \( x^4y^4 \) in \((x - 3y)^8\).

(4) (3 pts) How many 9-bit strings contain exactly two non-adjacent zeros?

For your reference, 110110111 is an example of a 9-bit string containing exactly two non-adjacent zeros.
(5) (4 pts) How many functions $f : X \rightarrow Y$ are there when $|X| = 5$ and $|Y| = 4$? How many of these functions are onto? 1 – 1?

(6) (4 pts) Prove that $C(2n, 2) = 2C(n, 2) + n^2$ where $n \geq 2$ is an integer.