Chapter 5:

(1) Consider

\[ \int_1^5 \sqrt{36 - x^2} \, dx. \]

Write down, but do not evaluate, the Right Riemann sum for this definite integral using \( n \) subintervals.

(2) Consider

\[ \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{(2 + \frac{8k}{n})^2} \cdot \frac{8}{n}. \]

Write down and evaluate a definite integral that this limit of Right Riemann sums represents. Answers can vary.
(3) Consider
\[ \int_0^2 x^2 \, dx. \]

Write down and evaluate the limit of Right Riemann sums for this integral.

You may want to use the following result:

\[ \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \]

(4) \[ \int_{-2}^{2} (2 - |x|) \, dx. \]

Hint: Use either geometry or symmetry.
(5) Find
\[ \int (x + 1)^3 \, dx. \]

(6) Find
\[ \int \frac{3x^3 - 2x^{-2}}{x} \, dx. \]

(7) Find
\[ \int_{0}^{\ln \sqrt{3}} \frac{e^x}{1 + e^{2x}} \, dx. \]
(8) Find
\[ \int_{0}^{\pi/4} \cos^2 2x \, dx. \]

(9) Find
\[ \int_{0}^{2} \sqrt{1 - (x-1)^2} \, dx. \]
Hint: Use geometry!

(10) Find the average value of \( f(x) = \tan x \) on \([0, \frac{\pi}{4}]\).
(11) Find
\[
\int_{-1}^{1} \sin (\sqrt{x}) \, dx.
\]

Hint: Use symmetry!

(12) Which of the statements are true for the function \( f(x) \) with the following graph? Select the best answer.

(a) \( \int_{0}^{6} f(x) \, dx = 0 \),

(b) \( -5 \leq \int_{0}^{3} f(x) \, dx \leq -3 \)

(c) The average value of \( f(x) \) on \([2, 5]\) is positive.

(d) All of the above.
(13) A function has to be differentiable for it to be integrable.

(a) True,
(b) False.

(14) Find
\[ \int_{0}^{\sqrt{5}} x\sqrt{x^2 + 1} \, dx. \]
(15) Let

\[ f(x) = \begin{cases} 4 - x^2 & \text{if } x < 2 \\ x - 2 & \text{if } x \geq 2 \end{cases} \]

Determine the integral:

\[ \int_{-2}^{4} f(x) \, dx. \]

(16) Find the following derivative in two ways: (a) By evaluating the integral first and then taking the derivative and (b) by applying the Fundamental Theorem of Calculus and the Chain Rule!

\[ \frac{d}{dx} \int_{-3}^{x^2} \frac{1}{4 + t} \, dt. \]
(17) Find the following derivative:

\[
\frac{d}{dx} \int_{\frac{1}{\sqrt{x}}}^{1} e^{-t^2} dt.
\]

Hint: Switch the order of integration and use both the Fundamental Theorem of Calculus and the Chain Rule.

(18) Find

\[
\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx.
\]
(19) Given that $2x^3 - 1 = \int_a^x f(t)dt$, find $f(t)$ and the lower limit $a$.

Hint: First take the derivative of both sides.

(20) Find

$$\int_0^5 x\sqrt{x + 4}dx.$$
(21) Find
\[ \int_{0}^{1} (x^n + \sqrt{x}) \, dx. \]

(22) Find \( c \) on \((1, 10)\) such that \( f(c) \) is the average value of \( f(x) = \frac{1}{x} \) on \([1, 10]\).
(23) Find\[ \int \frac{dx}{4 + 9x^2}. \]

Hint: Begin by force factoring the 4 out of the denominator. Then use an appropriate substitution.

(24)

The function \( f(x) = \int_0^x \sin(t) \, dt \) always increases for \( 0 < x < \pi \).

(a) True,
(b) False.

(25) If \( f(x) \) is an integrable function over \([a, b]\) then:

\[ \int_a^b |f(x)| \, dx = \left| \int_a^b f(x) \, dx \right|. \]

(a) True,
(b) False.
Chapter 6:

(1) Suppose the vertical velocity of an object, \( t \) seconds after launch, is given in meters by \( v(t) = -9.8t + 29.4 \). Find the change in height over the time interval \([0, 4]\). Find the total vertical distance traveled (up and down) over the time interval \([0, 4]\).

(2) Water flows out of a tank at the rate given by \( V'(t) = 9t + 1 \) ft\(^3\)/min. If the tank holds 54 cubic feet of water, then how long does it take to empty a full tank?

Hint: If \( V(t) = \) the amount of water that has flowed out at time \( t \), then:

\[
V(b) - V(0) = \int_0^b V'(t) \, dt.
\]
(3) Find the area in the first quadrant bounded by \( y = x^3 \) and \( y = x^{2/3} \).

(4) Find the area of the region in the first quadrant bounded by \( 2\sqrt{x} + \sqrt{y} = 2 \).

Hint: Solve for \( y \).
(5) Let $b > 1$ be a constant. Find the area bounded by $y = \frac{1}{x}$, $x = 1$, and $x = b$.

(6) Let $b > 1$ be a constant. Find the volume of revolution for revolving the area bounded by $y = \frac{1}{x}$, $x = 1$, and $x = b$ about the $x$-axis.
(7) Let $R$ be the region bounded by $y = \sqrt{2x - x^2}$ and $y = 0$. Using the disc method, find the volume of revolution for this region revolved about the $x$-axis.

(8) Using the shell method, write down, but do not evaluate, an integral for revolving the same region $R$ as in problem (7) about the $y$-axis.

Note: This integral can be done by hand by completing the square and using a trigonometric substitution.
(9) Let $R$ be the region in the first quadrant bounded by $y = 1 - \sqrt{x}$. Determine the volume of revolution of revolving $R$ about the line $x = -1$.

Hint: Use the shell method.

(10) Find the arc length of the curve $y = 2x^{3/2}$ on the interval $[0, 7]$. 
(11) Find the arc length of the curve $y = \ln(\sec x)$ on the interval $[0, \frac{\pi}{4}]$.

(12) Write down, but do not evaluate, an integral for the arc length of the curve $y = e^{-2x}$ on $[0, 5]$. 
(13) Suppose a thin rod that occupies the interval \([0, 1]\) has a linear density function 
\(\rho(x) = 3\sqrt{3x + 1}\) in kg/m. Find the mass of the rod.

(14) The force required to move a spring is directly proportional to the distance the spring 
stretches (or compresses) from its equilibrium position. Suppose it takes 196 N to 
stretch a large spring 2 meters from its equilibrium position. How much work is 
required to stretch the spring by 2.5 meters?
(15) A cylindrical water tank has a height of 5 meters and a radius of 1 meter. How much work is required to empty the tank from the top if it is full?

(16) A dam’s lower edge is in the shape of the parabola, $16y = x^2$, from $x = -8$ to $x = 8$. Determine the force on the dam if the water level is $y = 4$, that is, the dam is full. Assume the units here are meters.
(17) Find
\[ \int \frac{e^{-x}}{1 + e^{-2x}} \, dx. \]

(18) Find
\[ \int_{1}^{4} \frac{5 \sqrt{x}}{\sqrt{x}} \, dx. \]
(19) Find
\[ \int \frac{dx}{\sqrt{x}\sqrt{1-x}}. \]

Hint: Use either substitution \( u = \sqrt{x} \) or \( u = \sqrt{1-x} \).

(20) Suppose a population grows at a rate of 3 percent per year. If the population initially had 10 million individuals, then how long until the population has 30 million individuals.

Hint: \( y' = 0.03y \) so \( y = Ce^{kt} \)
Chapter 7:

(1) Find

\[ \int_0^1 \tan^{-1}(x) \, dx. \]

(2) Find

\[ \int x \ln(x) \, dx. \]
(3) Find
\[ \int e^{-t} \sin(t) dt. \]

(4) Find
\[ \int \frac{\sec^4 x}{\tan^2 x} dx. \]
(5) Find

$$\int \sin^2(x) \, dx.$$ 

(6) Find

$$\int \frac{dx}{(4 - 9x^2)^{3/2}}.$$
(7) Find
\[ \int_4^5 \frac{x + 5}{x^2 - 2x - 3} \, dx. \]

(8) Find
\[ \int \frac{x^2 - 2x}{2x^3 + x^2 + 2x + 1} \, dx. \]
(9) Find
\[ \int \frac{e^{2x}}{(1 + e^{4x})^{3/2}} \, dx. \]

(10) If \( a \) is a positive constant, then
\[ \int_{-a}^{a} \tan^{-1}(x^3) = 0. \]

(a) True,
(b) False.
(11) Find
\[ \int_1^{\infty} \frac{dx}{x^{3/2}}. \]

(12) Find
\[ \int_0^1 \frac{dx}{x^{2/3}}. \]
(13) Find
\[ \int_{-\infty}^{\infty} \frac{dx}{1 + x^2}. \]

(14) Show that the area under \( y = \frac{1}{\sqrt{x}} \) on \([1, \infty)\) diverges. Show also that this region’s volume of revolution about the \( x \)-axis on \([1, \infty)\) diverges.
(15) Which of the following statements are true?

(a) If a function is integrable on \([a, b]\) then it is differentiable on \((a, b)\).

(b) If the definite integral of \(f(x)\) is positive on \([a, b]\), then \(f(x)\) is sometimes positive on \([a, b]\).

(c) \[\int \tan x\,dx = \ln (\sec x) + C.\]

(d) All of the above.

(16) Solve the following differential equation:

\[y' = e^{-t} + y^2 e^{-t}.\]
(17) Find for $x > \ln 2$:

$$\int \frac{e^x}{e^{2x} - 4} \, dx$$
Chapter 5:

(1) \[ \sum_{i=1}^{n} \sqrt{36 - \left( 1 + \frac{4i}{n} \right)^2} \cdot \frac{4}{n} \]

(2) \[ \int_{2}^{10} \frac{1}{x^2} \, dx = \frac{2}{5} \]

(3) \[ \lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{2k}{n} \right)^2 \cdot \frac{2}{n} = \frac{8}{3} \]

(4) 4

(5) \[ \frac{1}{4} (x + 1)^4 + C \]

(6) \[ x^3 + \frac{1}{x^2} + C \]

(7) \[ \frac{\pi}{12} \]

(8) \[ \frac{\pi}{8} \]

(9) \[ \frac{\pi}{2} \]

(10) \[ \frac{2 \ln 2}{\pi} \]

(11) 0

(12) (d)
(13) 

(b)

(14) 

21

(15) 

\[ \frac{38}{3} \]

(16) 

\[ \frac{2x}{4 + x^2} \]

(17) 

\[ \frac{e^{-1/x}}{2\sqrt{x^3}} \]

(18) 

\[ \ln |e^x + e^{-x}| + C \]

(19) 

\[ f(t) = 6t^2; a = \sqrt[3]{\frac{1}{2}} \]

(20) 

\[ \frac{506}{15} \]

(21) 

1

(22) 

\[ \frac{9}{\ln 10} \]

(23) 

\[ \frac{1}{6} \tan^{-1} \left( \frac{3x}{2} \right) + C \]

(24) 

(a)

(25) 

(b)
Chapter 6:

(1) The change in height is 39.2 meters; the total distance traveled is 49 meters.

(2) $e^6 - 1$ minutes

(3) \[
\frac{7}{20}
\]

(4) \[
\frac{2}{3}
\]

(5) \[
\ln b
\]

(6) \[
\pi \left(1 - \frac{1}{b}\right)
\]

(7) \[
\frac{4\pi}{3}
\]

(8) \[
\int_0^2 2\pi x \sqrt{2x - x^2} \, dx
\]

(9) \[
\frac{13\pi}{15}
\]

(10) \[
\frac{1,022}{27}
\]

(11) \[
\ln (\sqrt{2} + 1)
\]

(12) \[
\int_0^5 \sqrt{1 + 4e^{-4x}} \, dx
\]

(13) \[
\frac{14}{3} \text{ kg}
\]
(14) \[ 306.25 \text{ J} \]

(15) \[ 122,500\pi \text{ J} \]

(16) \[ \frac{2,007,040}{3} \text{ N} \]

(17) \[ -\tan^{-1}(e^{-x}) + C \]

(18) \[ \frac{40}{\ln 5} \]

(19) \[ 2\sin^{-1}\sqrt{x} + C \]

(20) \[ \frac{100\ln 3}{3} \approx 37 \text{ years} \]
Chapter 7:

(1) \[ \frac{\pi}{4} - \frac{1}{2} \ln 2 \]

(2) \[ \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C \]

(3) \[ -\frac{1}{2} e^{-x} (\sin x + \cos x) + C \]

(4) \[ \tan x - \cot x + C \]

(5) \[ \frac{1}{2} \left( x - \frac{1}{2} \sin 2x \right) + C \]

(6) \[ \frac{x}{4\sqrt{4 - 9x^2}} + C \]

(7) \[ \ln \left( \frac{10}{3} \right) \]

(8) \[ \frac{1}{2} \ln |2x + 1| - \tan^{-1} x + C \]

(9) \[ \frac{e^{2x}}{2\sqrt{1 + e^{4x}}} + C \]

(10) \[ (a) \]

(11) \[ 2 \]

(12) \[ 3 \]

(13) \[ \pi \]
\[ \lim_{t \to \infty} (2\sqrt{t} - 2) = \infty; \quad \lim_{t \to \infty} \pi \ln t = \infty \]

(b)

\[ y = \tan (C - e^{-t}) \]

\[ -\frac{1}{2} \ln \left( \frac{e^x + 2}{\sqrt{e^{2x} - 4}} \right) + C \]

or by using properties of the logarithm

\[ \frac{1}{4} \ln \left( \frac{e^x - 2}{e^x + 2} \right) + C \]